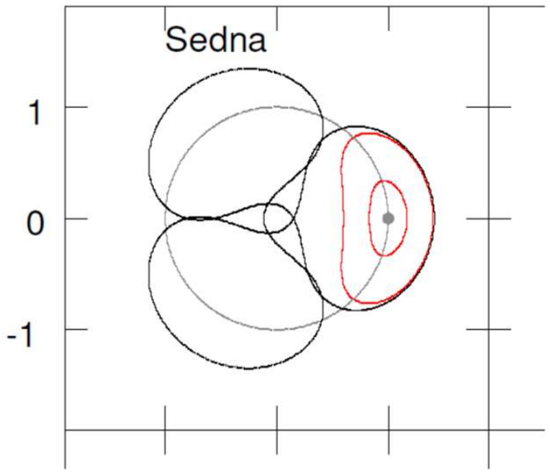

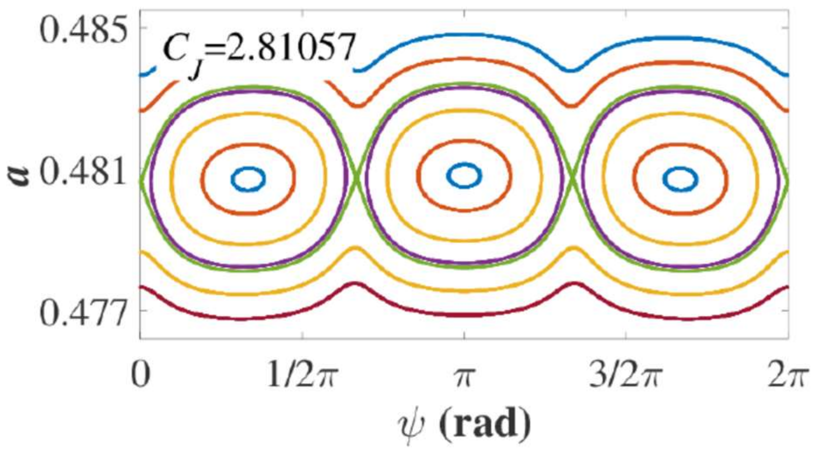

Vertiefungsmodul: Dynamik von Planeten und Planetesimalen

Corralling a Distant Planet with Extreme Resonant Kuiper Belt Objects

Presentation given by Bruno Reimann, 6430134
Vienna, June 22, 2017

Credit

Corralling a distant planet with extreme resonant Kuiper belt objects

The Astrophysical Journal Letters 824:L22 (6pp), 2016 June 20

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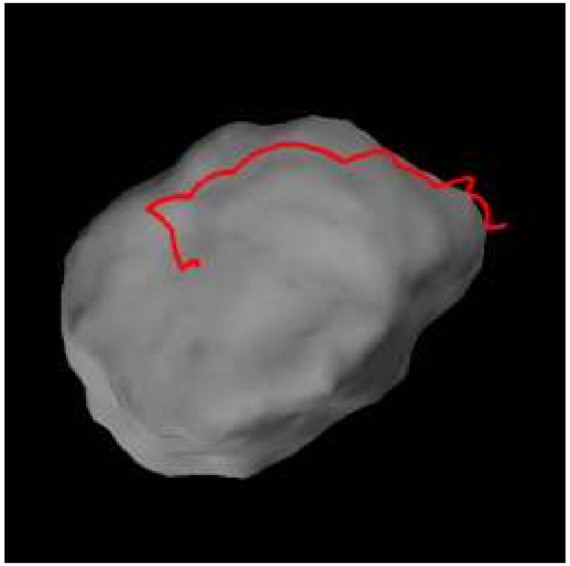

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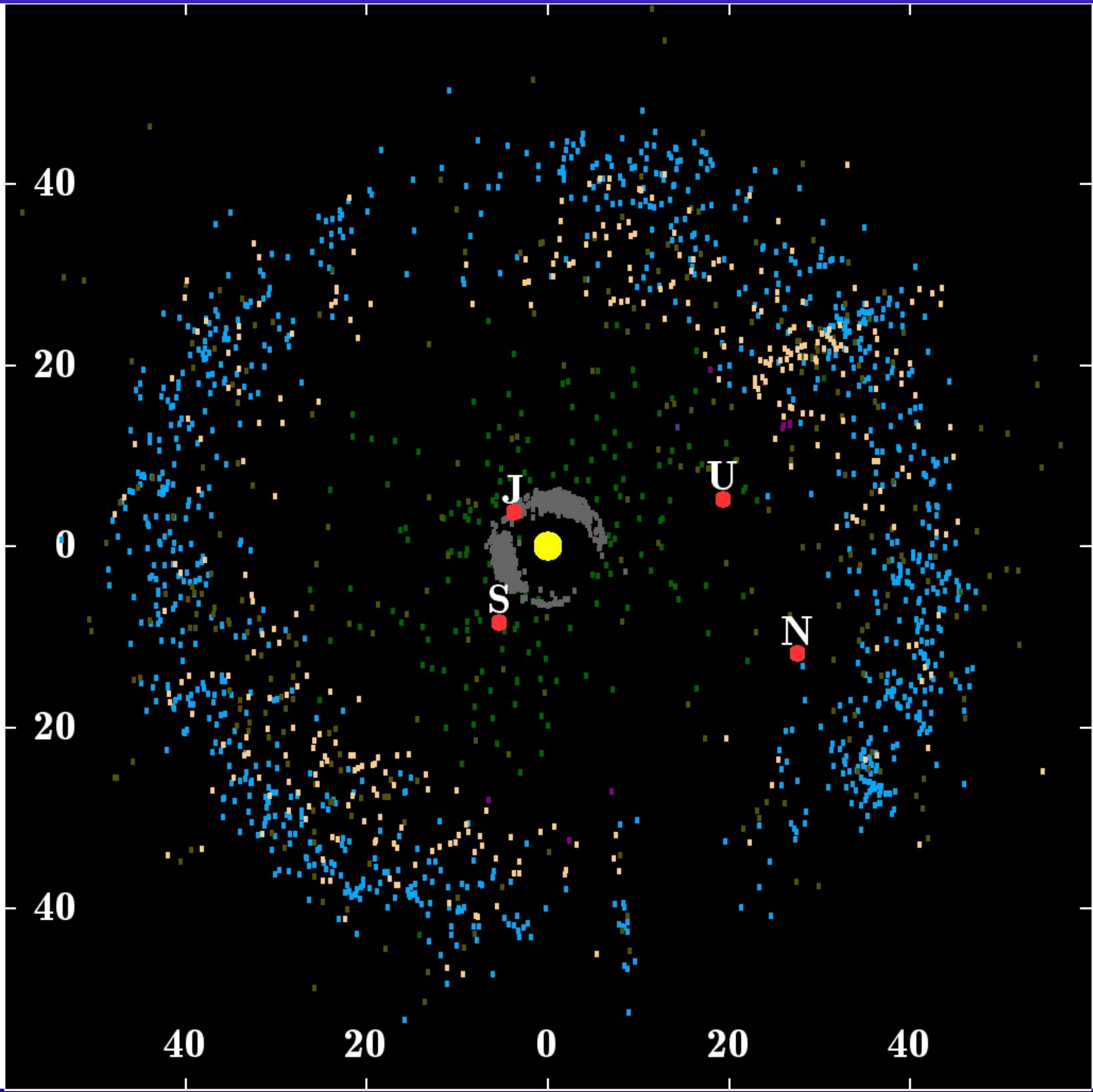
Kuiper Belt



- Circumstellar disc in the solar system
- 30 – 50 AU from the sun
- Consists of mainly small bodies, remnants from the solar system’s formation
- Kuiper Belt Objects (“**KBOs**”) are composed largely of frozen volatiles (“ices”): CH_4, NH_3, H_2O
- Since 2000 detection of large KBOs with diameters between ~500 and ~1500 km
- Most KBOs are significantly controlled by the gravity of the giant planets Jupiter, Saturn, Uranus, and Neptune

Initial Situation and Aim

- There are **six known KBOs** with semi major axes $a \geq 150$ and perihelion distances $q \geq 40$ au, unlikely to be perturbed by the Giant Planets.
  They are known as "detached objects", different from classical, resonant, and scattered/scattering KBOs; named extreme KBOs ("eKBOs"):

Sedna (90377)	2010 GB174	2004 VN112
2012 VP113	(148209)	2013 GP136

  Orbital inclinations to the ecliptic: 11°9 – 33°5
  Large orbital perturbations in their past history → very large orbital eccentricities: $e \geq 0{,}7$

- **Hypothesis**: orbital dynamics of eKBOs are dominated by gravitational perturbations of a planet ("**HP**") ("**extant distant planet hypothesis**")

distance several hundred au	mass ~10 $M_{\oplus}$
semi major axis a ~700 au	eccentricity e ~0,6

  large uncertainties!

- **Aim of this paper**: narrow the range of HP's parameters in the framework of the extant distant planet hypothesis

Structure

Mean motion resonances

Resonant orbital geometry constraints

Planet Mass

Planet's current location in the orbit

Orbits and Mean Motion Resonances

Ordering the six eKBOs according to decreasing orbital periods (Sedna is #1):

j	2	3	4	5	6
P_1 / P_j	1,596	1,993	2,666	3,303	6,115
≈	8/5	2/1	8/3	10/3	6/1

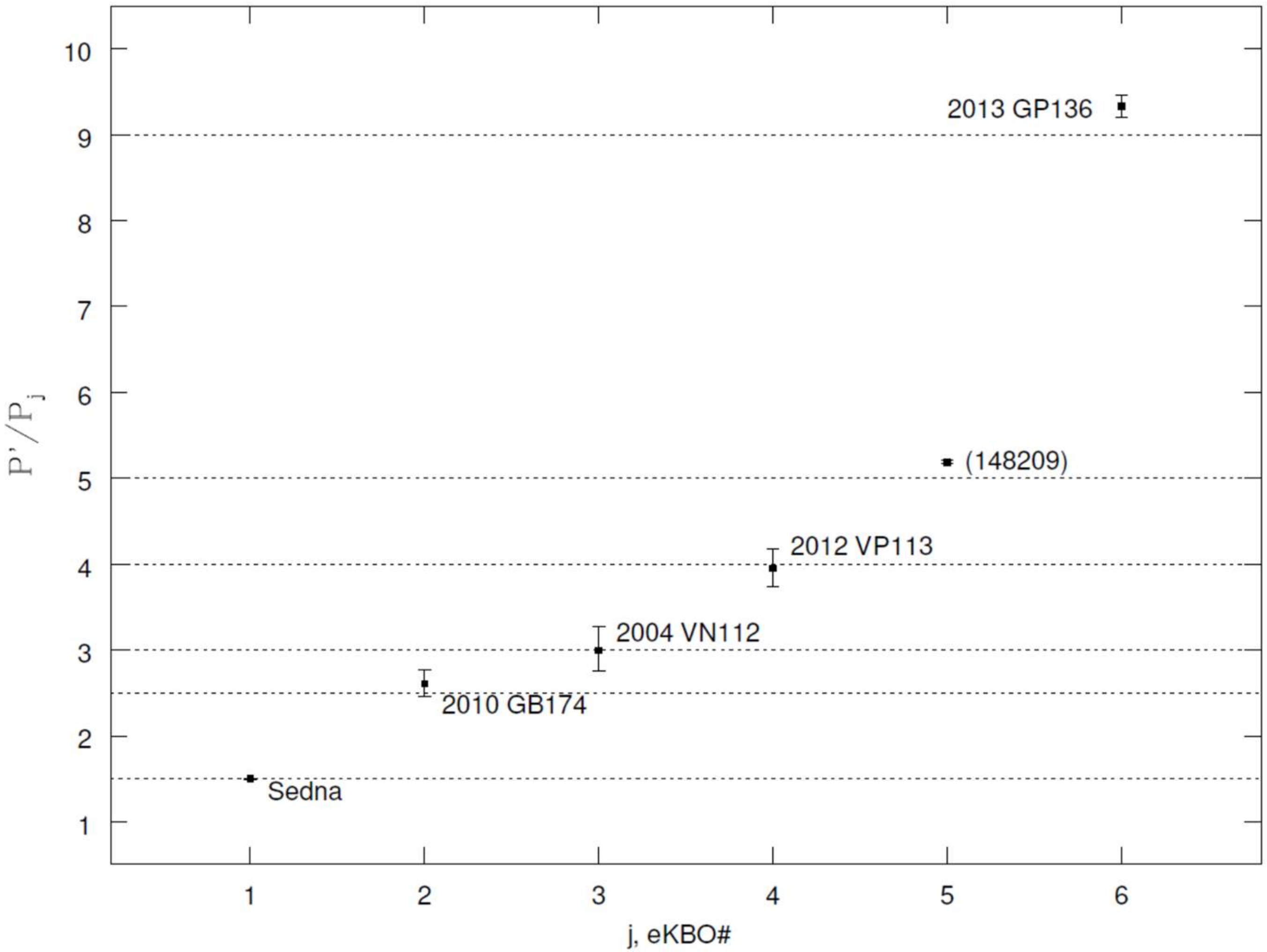
Computing the best-fit orbits for the six eKBOs including uncertainties (software by Bernstein & Khushalani, 2000)

j	1	2	3	4	5	6
a	506,84	350,7	319,6	265,6	221,59	149,84
σ(a)	0,51	4,7	6,0	3,3	0,16	0,47

Simple assumptions for HP's resonant orbit:

- HP's orbital period P' is a little longer than P_1 (→ strong gravitational effects)
- Variation of P'/P1: 2/1, 3/2 or 4/3

Mean Motion Resonances



Period ratios of the six eKBOs with the hypothetical planet. Error bars: 3σ.

The orbit period P' of HP is assumed to be 3/2 times Sedna's best-fit orbital period, P_1 = 11,411 years.

Dotted horizontal lines: rational ratios 3/2, 5/2, 3/1 4/1, 5/1 and 9/1.

Resonant Orbital Geometry Constraints

Stable resonant orbits of the eKBOs:

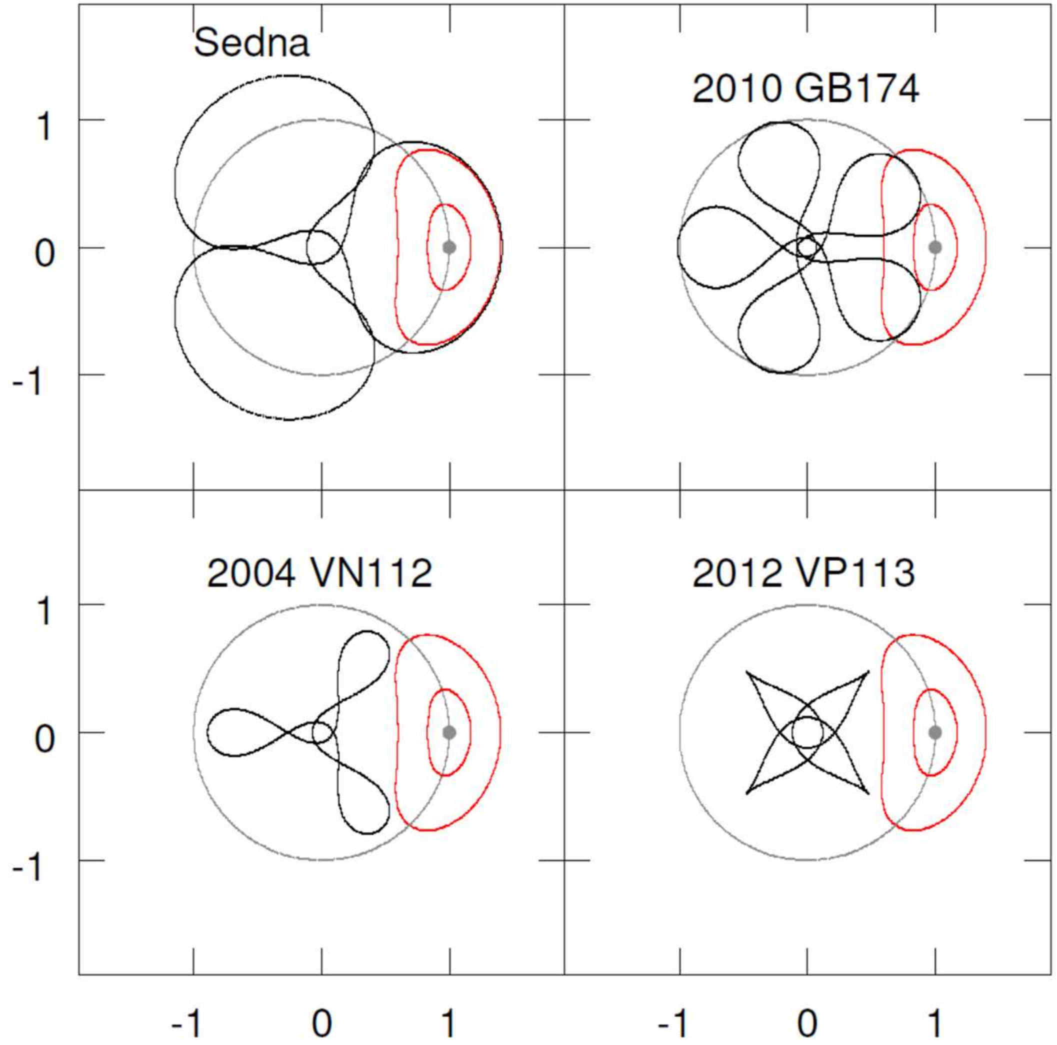
- will avoid close approaches with HP,
- will have a discrete number of geometries for the conjunctions of the eKBO with the planet.

These are described by librations of the critical resonant angle,

$$\phi = (p+q)\lambda' - p\lambda - q\varpi.$$

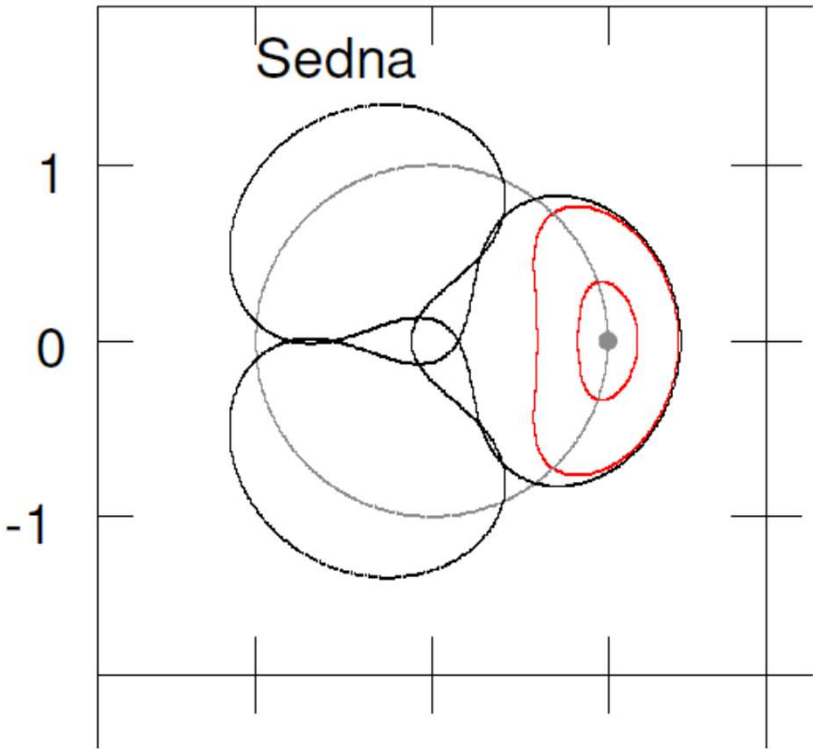
p, q: integers, λ: mean longitudes, ϖ: longitudes of perihelion
prime (‘): orbital parameters of HP

Resonant Orbital Geometry Constraints



- Frame: co-rotating with the mean angular velocity of HP
- Black: resonant orbits in the rotating frame, zero libration
- Red loops: trace of HP in possible eccentric orbits of eccentricity 0.17 and 0.4
- Distances scaled to the semi major axis of HP
- The grey circle is of unit radius

Resonant Orbital Geometry Constraints

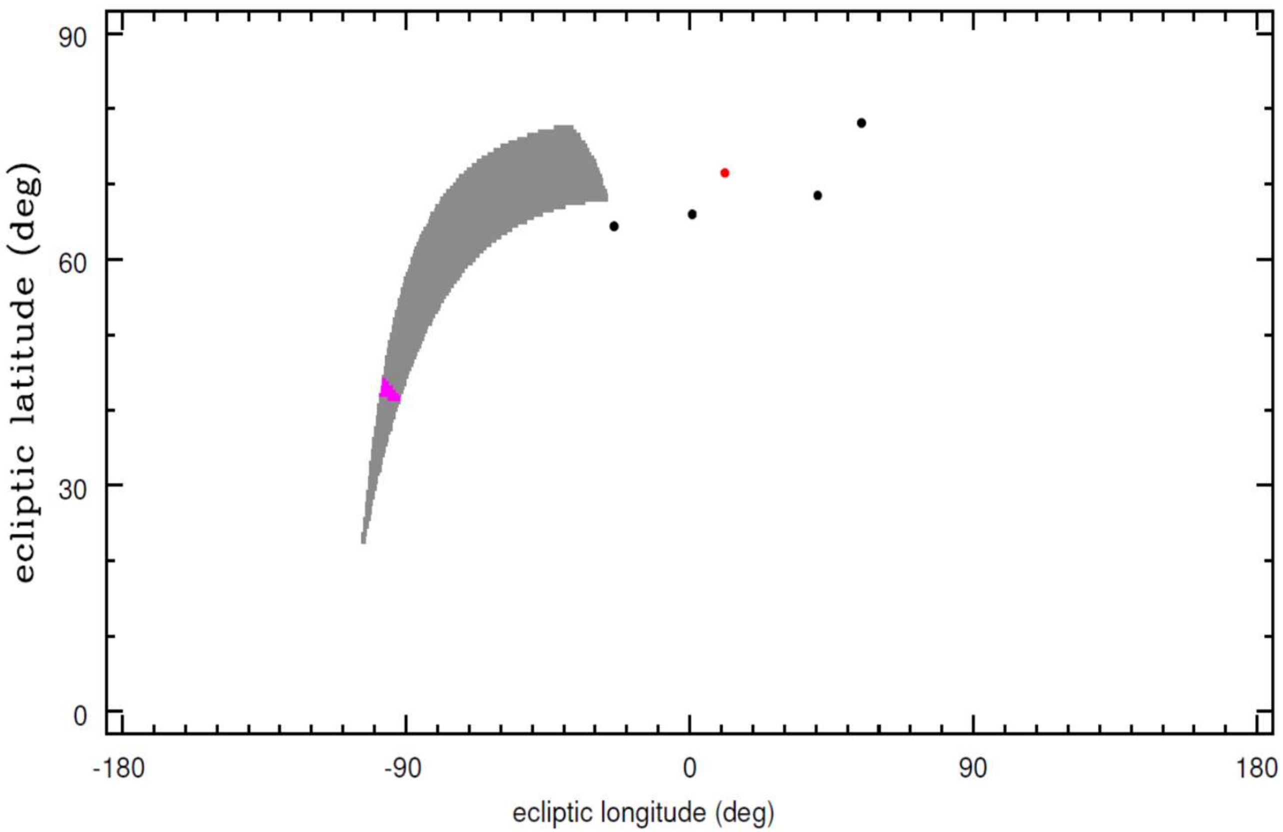


- For Sedna, stable libration is likely near $\Phi = \pi$ and unlikely near $\Phi = 0$ (small encounter distances)
- → conjunctions with HP occur near one of 3 longitudes, $\pm\pi/3$ and π
- → long term stability of Sedna would limit e‘ = e(HP) $\leq$ 0,41
- → the orbit of 2004 VN112 limits e‘ $\leq$ 0,41 for $\Phi = \pi$

Strict coplanarity of HP is possible for only one eKBO since eKBO’s are not coplanar

→ →

Ecliptic longitude and latitude of orbit normal vectors



- Black dots: orbit planes, as described by the ecliptic latitude and longitude of the orbital normal vectors, of the four potentially resonant eKBOs.
- Red dot: indicates their mean plane.
- The region shaded in grey identifies the planes for which the four eKBOs have g values within 45° of $\pm\pi/2$.

- The region shaded in magenta identifies the subset for which the relative inclinations of the eKBOs are also within 10 degrees of the stationary inclination values at the periodic orbit of the third kind (Poincaré / a class of periodic orbits of the three dimensional restricted three body problem).

Arguments of perihelion

Arguments of perihelion, g, relative to the main plane of the four longest period eKBOs:

145°4	-70°4	12°0	-39°3

Planet Mass (in short)

- Tentative planet mass ~10 $M_{\oplus}$ defines resonance widths comparable to uncertainties of semi major axes of observed eKBOs.

- Resonant libration periods would be in the order of a few 10^6 years, much shorter than the age of the solar sytem.

→→ 10 $M_{\oplus}$ is approximately the minimum mass necessary to maintain periodic orbits of the third kind for the 4 eKBOs.

Periodic orbits of the third kind

PERIODIC ORBITS OF THE THIRD KIND IN THE RESTRICTED THREE-BODY PROBLEM WITH OBLATENESS

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(Received 2 October, 1989)

Abstract. By use of the secular perturbing potential due to oblateness, the existence of periodic orbits of the third kind is established through Poincaré's method of analytic continuation using Delaunay's canonical variables and other three sets of canonical variables which are linear combinations of Delaunay's variables, in the three-dimensional restricted three-body problem when the more massive primary is an oblate spheroid with its equatorial plane coincident with the plane of motion. For two sets of the canonical variables, the singularities are found at the inclinations $i = 68\overset{\circ}{.}5833$, $111\overset{\circ}{.}4167$, while for the other two sets of the canonical variables, the singularities are at $i = 55\overset{\circ}{.}3854$, $103\overset{\circ}{.}575$.

Astrophysics and Space Science (ISSN 0004-640X), vol. 166, no. 2, April 1990, p. 211-218.

Planet's Current Location in its Orbit

- **Libration**: Each resonant eKBO constrains the longitude of HP: ← critical resonance angle librates about 0° or 180°

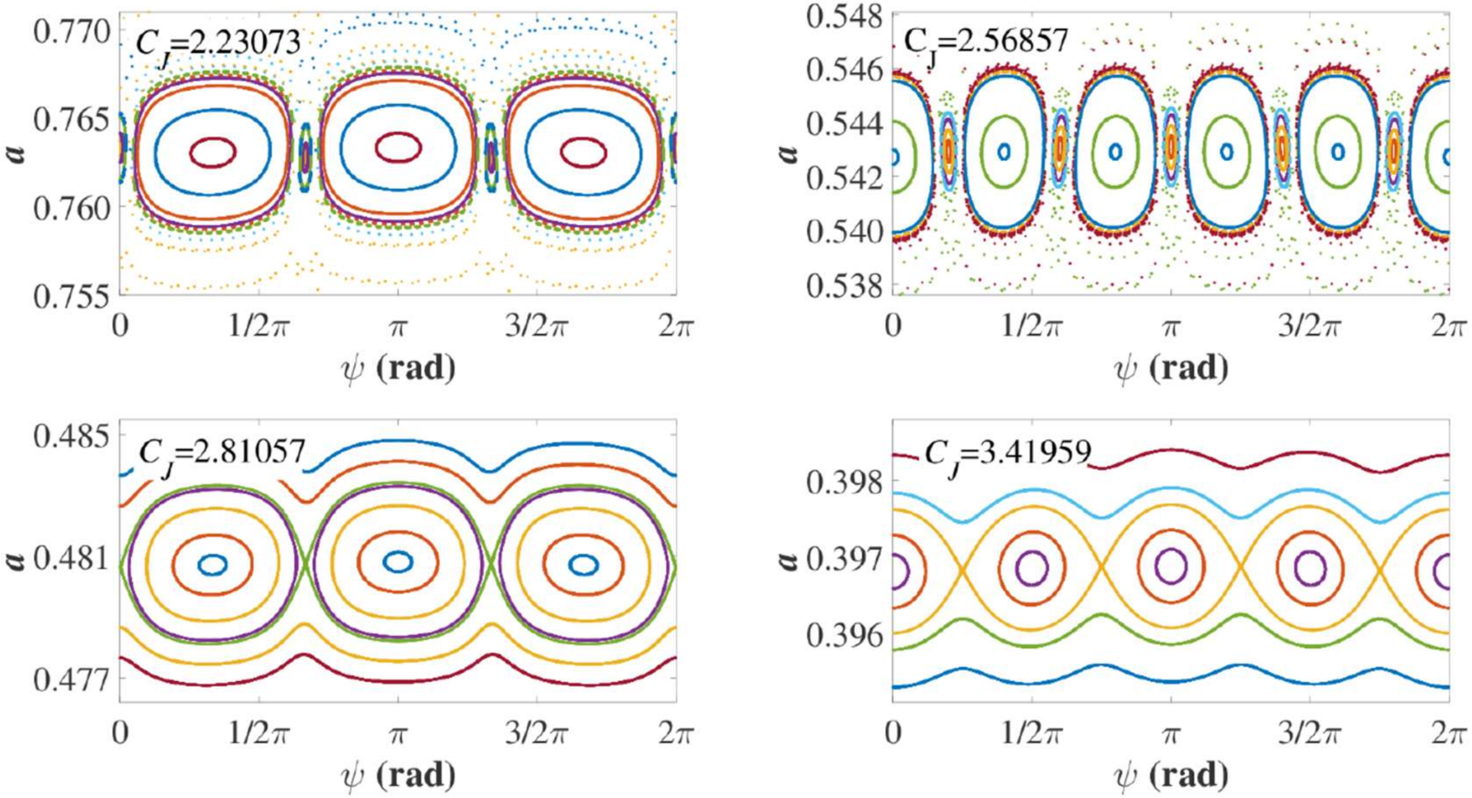
$$\phi = (p+q)\lambda' - p\lambda - q\varpi.$$

- **Example**: libration of Sedna.
  Libration of Φ about 180° for Sedna's 3/2 resonant orbit.
  Assuming that Sedna is presently near its perihelion, $\lambda = \varpi$
  → HP can be in proximity of 60°, 180° or 300° from Sedna's longitude of perihelion ϖ. Libration of Φ would allow HP to be located as far as ~54° from these longitudes
  Only 3 small zones of ±6° around ϖ, ϖ + 120° and ϖ - 120° are excluded for HP's present location.

- Must take into account **HP's gravity**.
  Compute **surface of sections** for the circular planar restricted three body problem, varying HP's mass from 10^{-6} to 10^{-3} $M_\odot$, for the neighborhood of 3/2, 5/2, 3/1 and 4/1 MMRs and the extreme eccentricities of the 4 eKBOs.

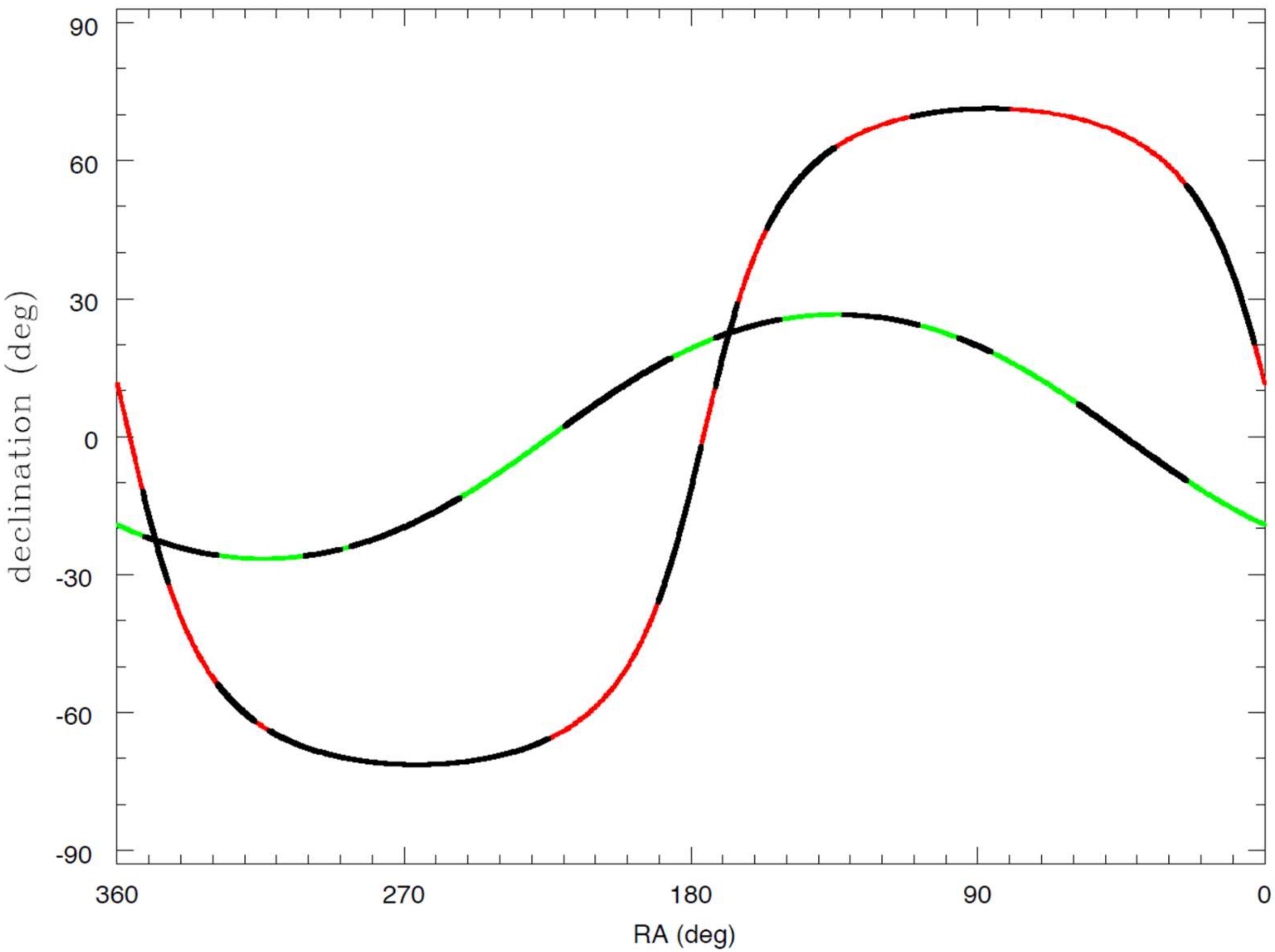
→ →

Surface of sections



Surfaces of section for the circular planar restricted three body problem, for mass ratio $m_2/(m_1 + m_2) = 3x10^5$, and Jacobi constant values as indicated. These are phase space portraits in the vicinity of the 3/2, 5/2, 3/1 and 4/1 resonant orbits of eccentricity 0.85, 0.87, 0.85 and 0.7, respectively. The abscissa, ψ, measures the angle between the planet and the particle's pericenter; the ordinate is the particle's barycentric semi major axis, in units of the circular orbit radius of the planet.

Possible locations of HP in the sky



The possible locations of HP in the sky: in **green** for a representative low inclination orbit, in red for a representative high inclination orbit; the zones in **black** are excluded by the resonant eKBOs. A range of at least several degrees near each of these representative planes is allowed.

~ half of the ecliptic longitudes are excluded.

Summary

- The four known longest period KBOs (Sedna (90377), 2010 GB174, 2004 VN112, 2012 VP113) could all be in mean motion resonances with an unseen distant planet. The set of resonances is not uniquely determined.
- Assumption of a hypothetical planet of orbit period $P' \approx 17{,}117$ years (semi major axis $a' \approx 665$ au), which would have dynamically significant N/1 and N/2 period ratios with these four eKBOs.
- A broad sketch is given of how resonant geometries and resonant dynamics of these high eccentricity eKBOs help narrow the range of planet parameters; many details in this analysis would benefit from deeper investigation in future work and potentially improve planet parameters further.
- Calculations suggest two possibilities for the planet's orbit plane: a plane moderately close to the ecliptic and near the mean plane of the four eKBOs ($i \approx 18°$, $\Omega \approx 101°$), or an inclined plane near $i \approx 48°$, $\Omega \approx -5°$. The former offers dynamical stability of the four eKBOs by means of libration of the critical resonant angles, and the latter offers enhanced dynamical stability by means of additional libration of the argument of perihelion associated with periodic orbits of the third kind.

Summary (cont.)

- A planet mass of $\approx 10\ M_\oplus$ defines resonance widths that are similar to or exceed the semi major axis uncertainties of the observed eKBOs.
- Resonant libration periods would be on the order of a few megayears, much shorter than the age of the solar system. In the case of the inclined planet orbit, a planet of
  $\geq 10\ M_\oplus$ also supports librations about the periodic orbits of the third kind, with libration periods shorter than the age of the solar system. Both these estimates support the conjecture that the period ratio coincidences are of dynamical significance.
- Exclusion zones of the current location of the planet in its orbital path are determined, based on the MMRs of all four eKBOs; these exclude just over half of the orbital path, assuming a $10\ M_\oplus$ planet.
- Analysis supports the distant planet hypothesis, but should not be considered denitive proof of its existence.

Thanks for not snoring !