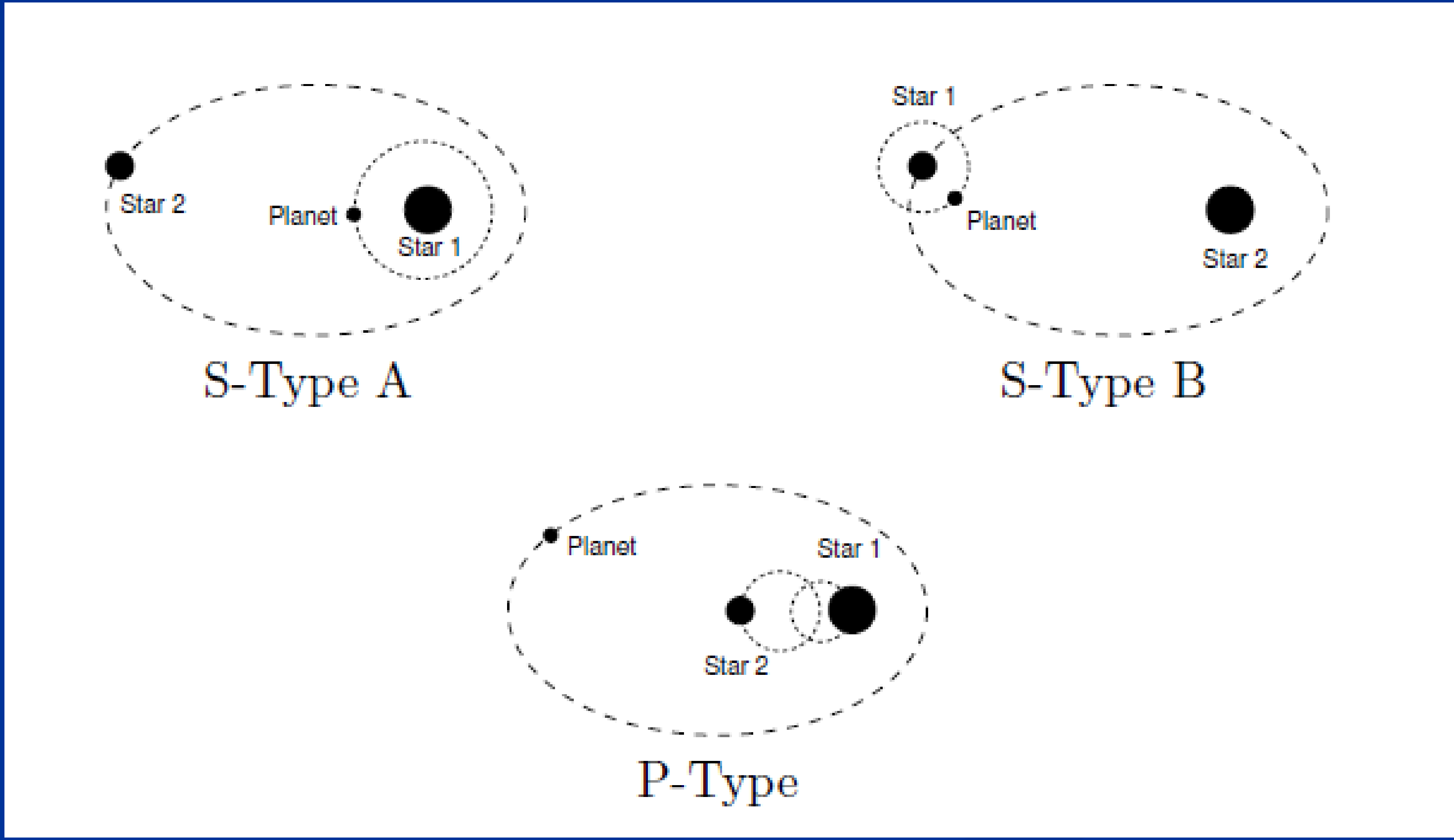
Planetary Motion in
Binary Star Systems

Outline

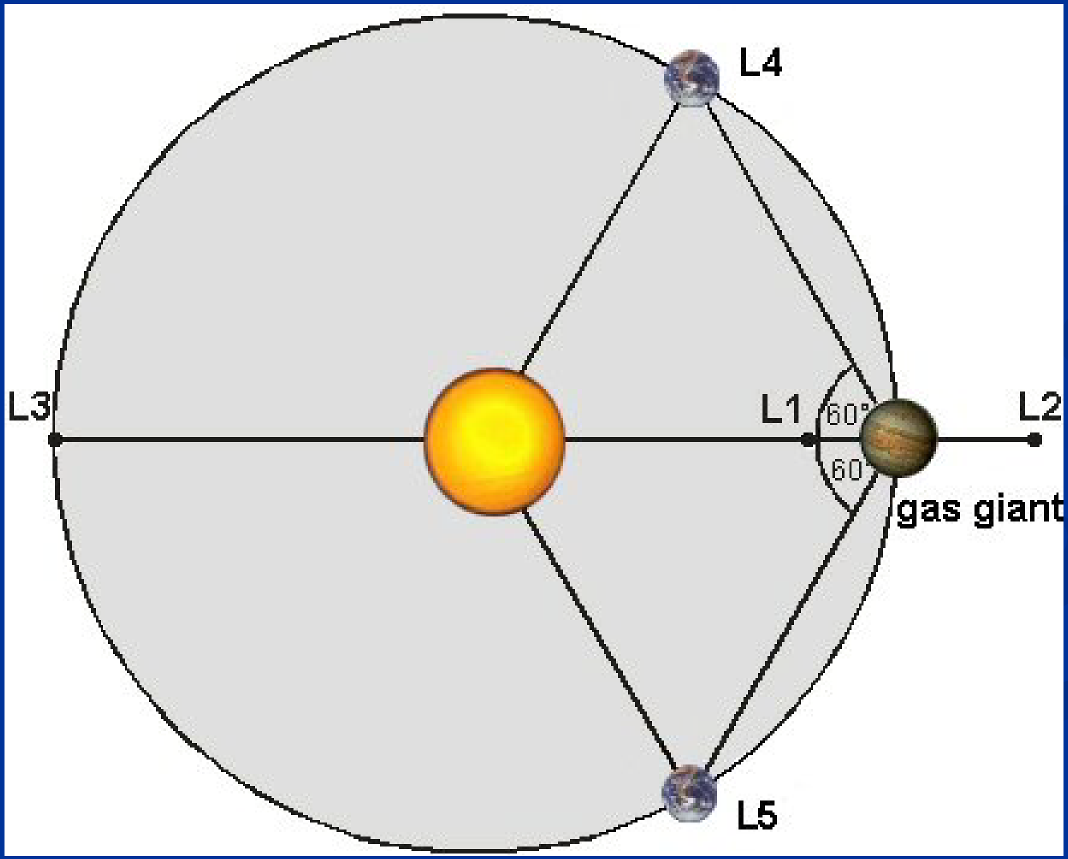
- Types of planetary motion in binary star systems
- Stability of planetary motion
- Examples of planetary motion in tight binaries
- Habitable planets in binaries?

Planetary motion in binaries:



Trojans – type motion
Near L4 and L5

L_4 and L_5 (stable for μ < 1:26) are at the third vertex of an equilateral triangle (Sun-Jupiter-Asteroid)



Studies by : Harrington (1977)
Graziani & Black (1981)
and Black (1982)

planar case:

No serious threat to the long-term stability

if the star is at a distance > 5 x planetary distance

inclined case:

at high i_{Binary} → the planetary orbits becomes unstable → Kozai mechanism

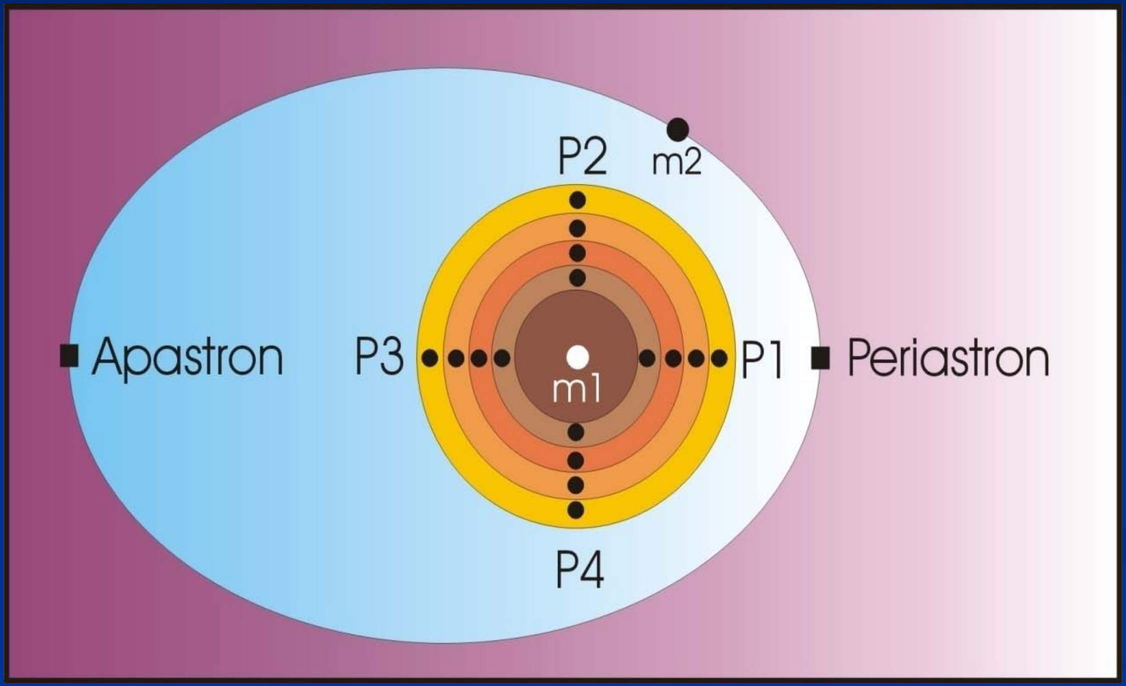
Pendleton & Black (1983)

Some General Numerical Studies of planetary motion in binary systems:

- Dvorak R.(1984 and 1986)
- Rabl & Dvorak R. (1988)
- Dvorak R., Froeschle C. & Froeschle Ch. (1989)

- Holman M. & Wiegert P. (1999)
- Pilat-Lohinger E. & Dvorak R. (2002)
- Szenkovits F. & Mako Z. (2008)

Stability analysis



Initial Conditions:
abinary **=1 AU**
ebinary**=[0, ….0.9]**

aplanet**=[0.1, …0.9]**
eplanet**=[0,… 0.9]**
i, 'Ω,ω, = 0°
M= 0°,90°,180°,270°

Numerical Methods:

Long-term numerical integration:

Stability-Criterion:

No close encounters within the Hill‘ sphere

(i)Escape time

(ii) Study of the eccentricity: maximum eccentricity

Chaos Indicators:

Fast Lyapunov Indicator (FLI)

C. Froeschle, R.Gonczi, E. Lega (1996)

MEGNO

RLI

Helicity Angle

Lyapunov Exponent

General Studies of S-type motion in binary systems:

- Dvorak R.(1984 and 1986)
- Rabl & Dvorak R. (1988)
- Dvorak R., Froeschle C. & Froeschle Ch. (1989)

  (200-500 periods)

- Holman M. & Wiegert P. (1999)

  (mass-ratios: 0.1-09, e_{Binary}: 0 – 0.9; 10000 periods)

Pilat-Lohinger & Dvorak (2002):

The Fast Lyapunov Indicator (FLI)

(see Froeschle et al., CMDA 1997)

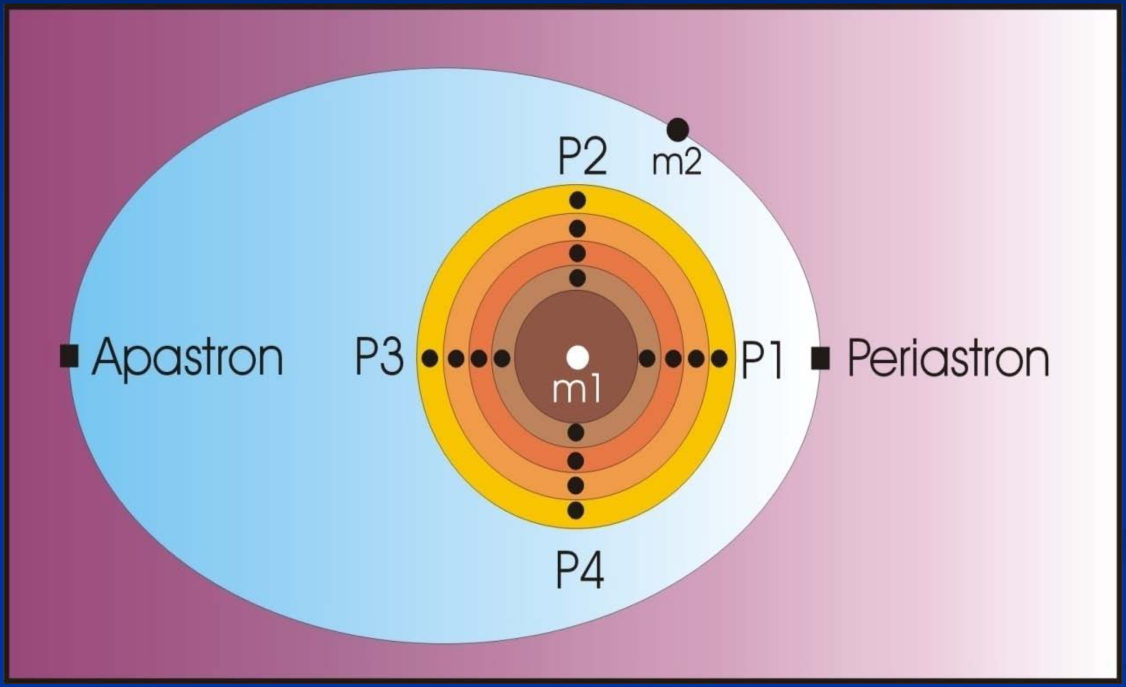
distinguish between regular and chaotic motion ;
length of the largest tangent vector:

$$FLI(t) = \sup_i |v_i(t)| \qquad i=1,\ldots,n$$

(n denotes the dimension of the phase space)

chaotic orbits can be found very quickly because of the exponential growth of this vector in the chaotic region;
For most chaotic orbits only a few number of primary revolutions is needed to determine the orbital behavior.

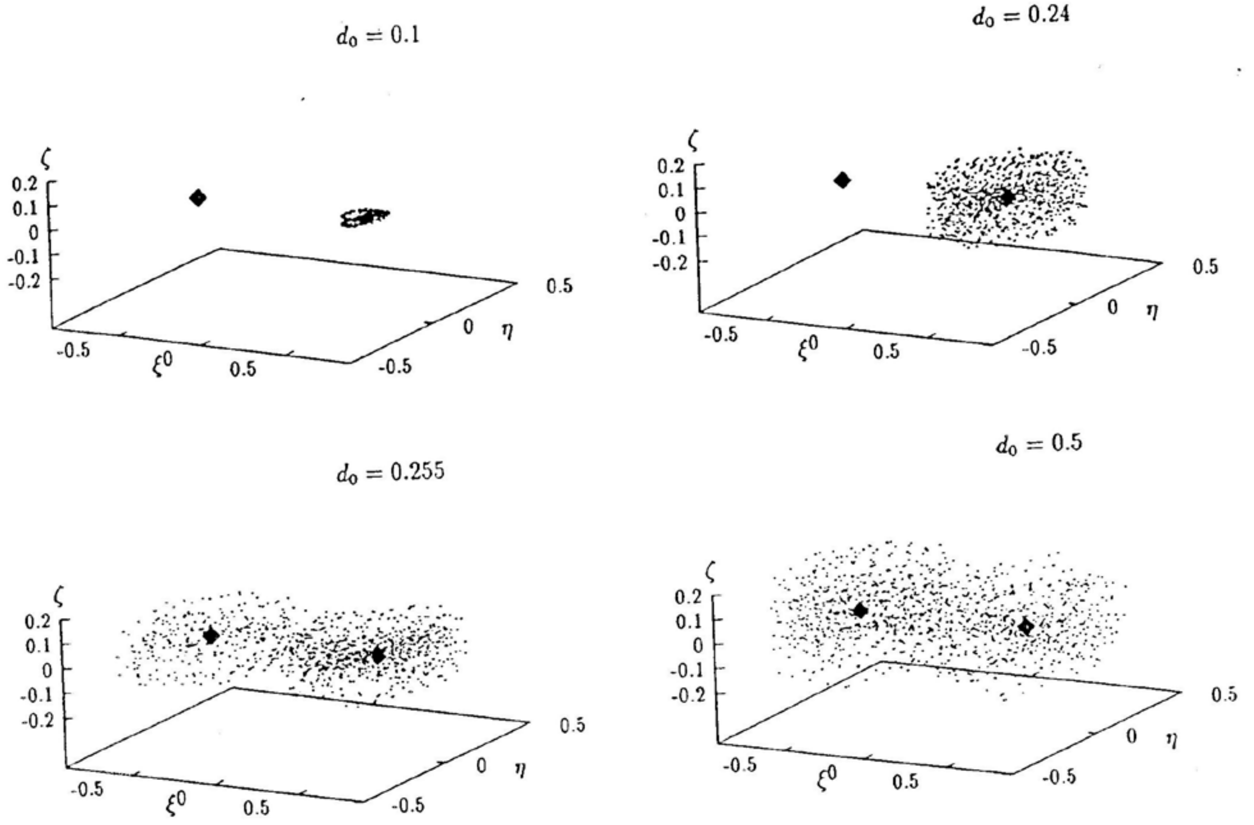
Stability analysis



Initial Conditions:
abinary **=1 AU**
ebinary**=[0, ….0.9]**

aplanet**=[0.1, …0.9]**
eplanet**=[0,… 0.9]**
i, 'Ω,ω, = 0°
M= 0°,90°,180°,270°

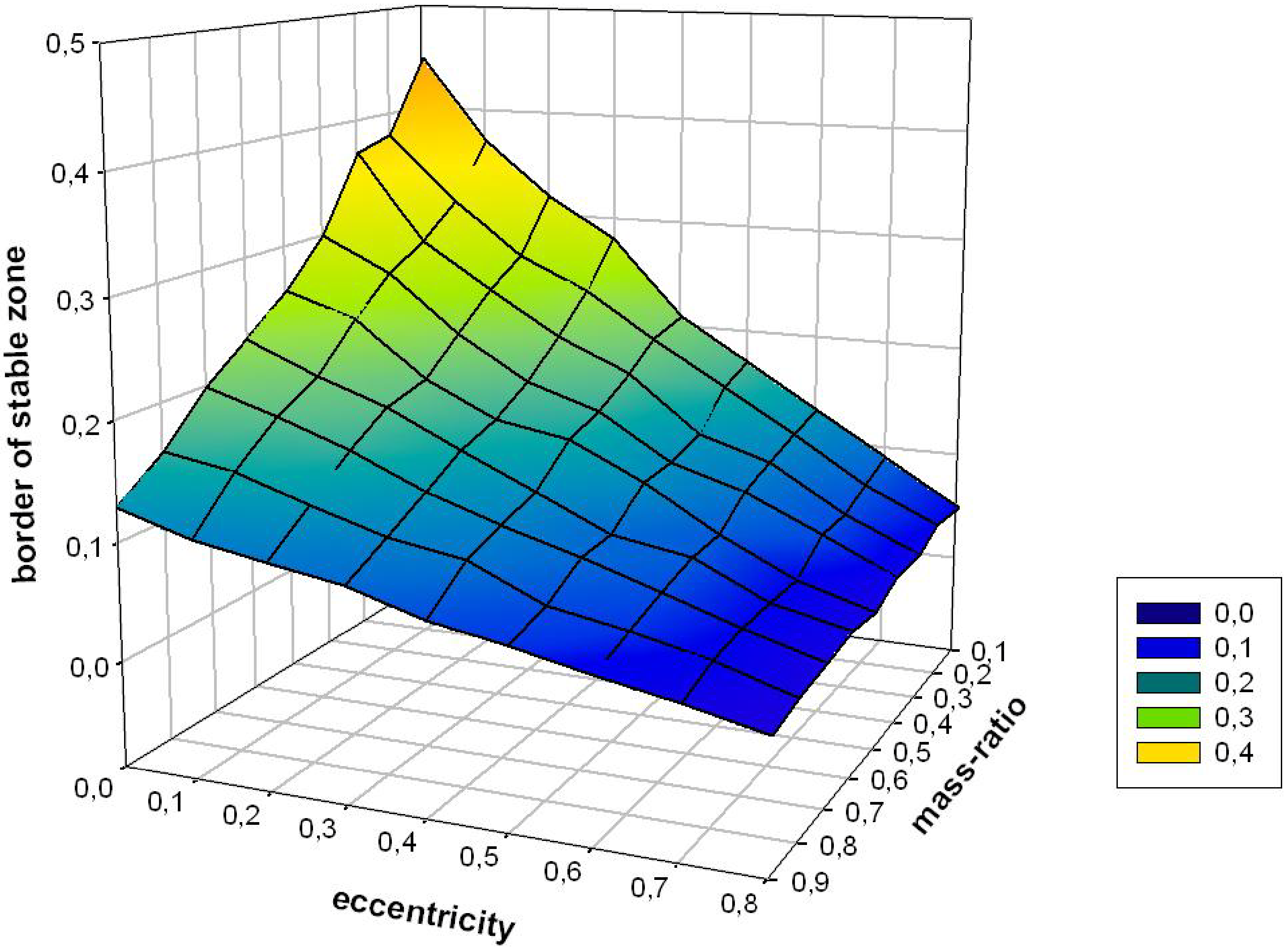
mass-ratio = 0.5



S-type motion

e_binary	mass-ratio 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0	0.45	0.38	0.37	0.30	0.26	0.23	0.20	0.16	0.13
0.1	0.37	0.32	0.29	0.27	0.24	0.20	0.18	0.15	0.11
0.2	0.32	0.27	0.25	0.22	0.19	0.18	0.16	0.13	0.10
0.3	0.28	0.24	0.21	0.18	0.16	0.15	0.13	0.11	0.09
0.4	0.21	0.20	0.18	0.16	0.15	0.12	0.11	0.10	0.07
0.5	0.17	0.16	0.13	0.12	0.12	0.09	0.09	0.07	0.06
0.6	0.13	0.12	0.11	0.10	0.08	0.08	0.07	0.06	0.045
0.7	0.09	0.08	0.07	0.07	0.05	0.05	0.05	0.045	0.035
0.8	0.05	0.05	0.04	0.04	0.03	0.035	0.03	0.025	0.02

Stable zone (in units of length) of S-type motion for all computed mass-ratios and eccentricities of the binary. The given size for each (mu,e_binary) pair is the lower value of the studies by Holman &Wiegert (AJ,1999) and Pilat-Lohinger &Dvorak (CMDA, 2002)



border of stable zone
0,5
0,4
0,3
0,2
0,1
0,0
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
eccentricity
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
mass-ratio
0,0
0,1
0,2
0,3
0,4

S-type motion

$$a_c = [(0.464 \pm 0.006) + (-0.380 \pm 0.010)\mu + (-0.631 \pm 0.034)e + (0.586 \pm 0.061)\mu e + (0.150 \pm 0.041)e^2 + (-0.198 \pm 0.074)\mu e^2]a_b \, ,$$

Least square fit published by Holman & Wiegert (1999)

Influence of the eccentricity of the planet?

Gamma Cephei: a_bin~ 20 AU; e_bin=0.4

e_plan	RTBP	1Mj	3Mj	5Mj	8Mj
0.0	0.20	0.20	0.20	0.20	0.20
0.1	0.20	0.20	0.20	0.19	0.19
0.2	0.18	0.18	0.17	0.16	0.16
0.3	0.17	0.16	0.16	0.15	0.15
0.4	0.17	0.15	0.14	0.13	0.13
0.5	0.15	0.12	0.11	0.09	0.09

Influence of the planet‘s mass ?

Is the elliptic restricted three body problem a good model ?

gamma Cephei

a_bin ~ 20 AU, e_bin ~ 0.4

e_plan	RTBP	1Mj	3Mj	5Mj	8Mj
0.0	4.0	4.0	4.0	4.0	4.0
0.1	4.0	4.0	4.0	3.8	3.8
0.2	3.8	3.6	3.4	3.2	3.2
0.3	3.4	3.6	3.2	3.0	3.0
0.4	3.4	3.0	2.8	2.6	2.6
0.5	3.0	2.4	2.2	1.8	1.8

Stability limits

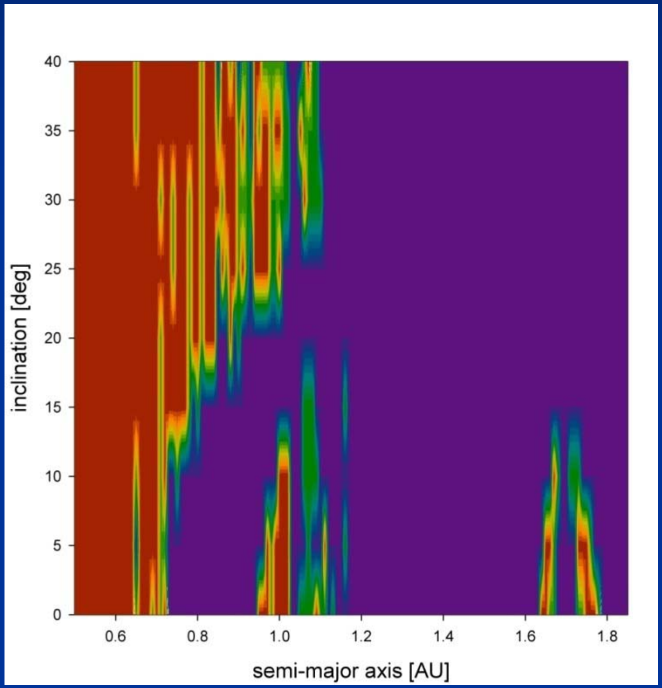
	mass-ratio	border of stable motion
Gamma Cephei:	0.2	3.2 – 3.8 AU
HD41004 AB:	0.36	(e=0.39) 4.6 – 2.8 AU
		(e=0.5) 4.6 – 2.53AU
Gliese 86:	0.42	close-in planet

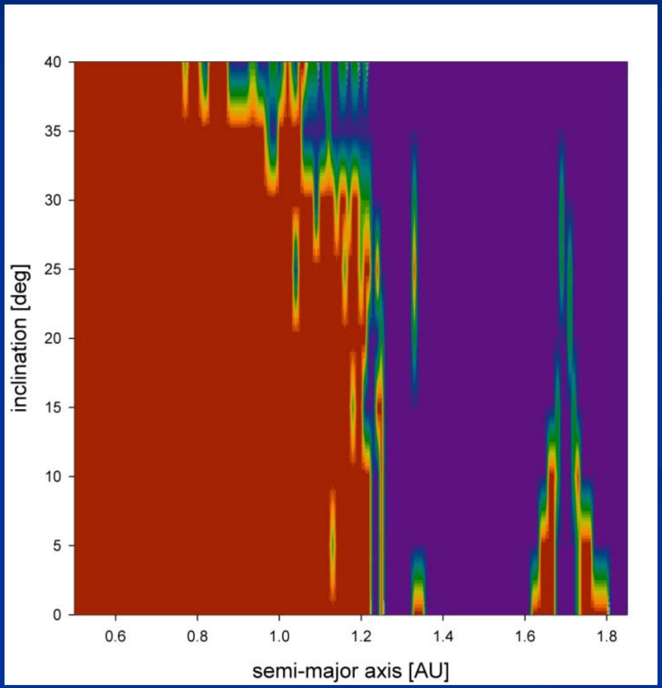
Influence of the secondary

on the planetary motion ?

with secondary

without secondary





gammaCep

e_b=0.44

e_pl=0.209

Influence of the secondary on the HZ?

THE ASTROPHYSICAL JOURNAL, 752:74 (11pp), 2012 June 10 doi:10.1088/0004-637X/752/1/74

AN ANALYTIC METHOD TO DETERMINE HABITABLE ZONES FOR S-TYPE PLANETARY ORBITS IN BINARY STAR SYSTEMS

SIEGFRIED EGGL[1], ELKE PILAT-LOHINGER[1], NIKOLAOS GEORGAKARAKOS[2], MARKUS GYERGYOVITS[1], AND BARBARA FUNK[1]
[1] Institute for Astronomy, University of Vienna, Türkenschanzstr. 17, A-1180 Vienna, Austria; siegfried.eggl@univie.ac.at, elke.pilat-lohinger@univie.ac.at
[2] 128 V. Olgas str., Thessaloniki 546 45, Greece
Received 2012 January 31; accepted 2012 April 10; published 2012 May 25

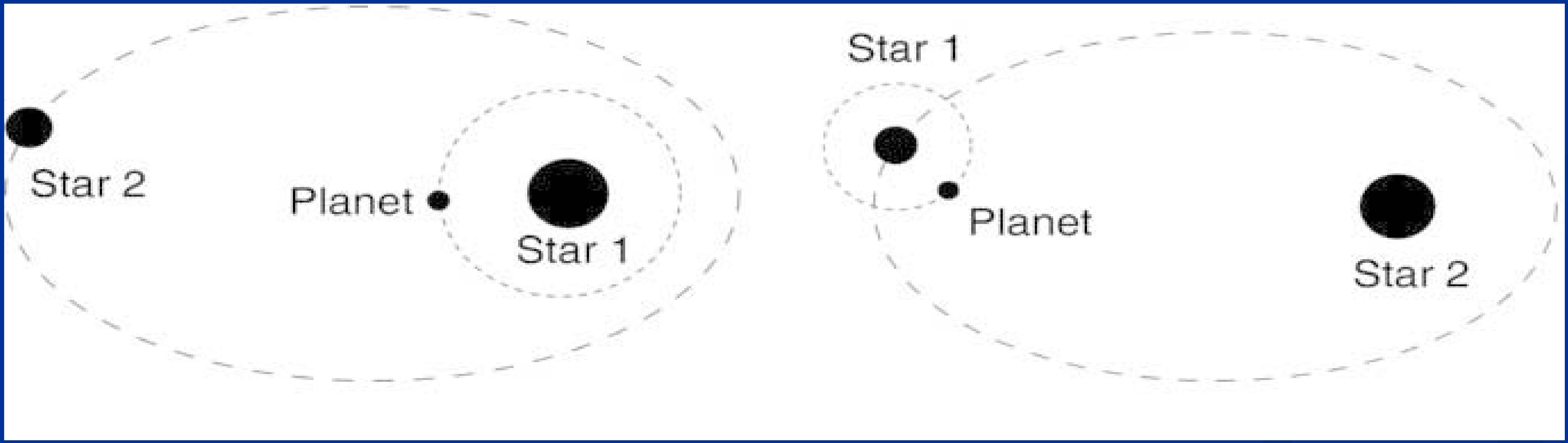
ABSTRACT

With more and more extrasolar planets discovered in and around binary star systems, questions concerning the determination of the classical habitable zone have arisen. Do the radiative and gravitational perturbations of the second star influence the extent of the habitable zone significantly, or is it sufficient to consider the host star only? In this article, we investigate the implications of stellar companions with different spectral types on the insolation a terrestrial planet receives orbiting a Sun-like primary. We present time-independent analytical estimates and compare them to insolation statistics gained via high precision numerical orbit calculations. Results suggest a strong dependence of permanent habitability on the binary's eccentricity, as well as a possible extension of habitable zones toward the secondary in close binary systems.

Key words: astrobiology – celestial mechanics – methods: analytical – planet–star interactions

Online-only material: color figures

Binary Star Configurations:

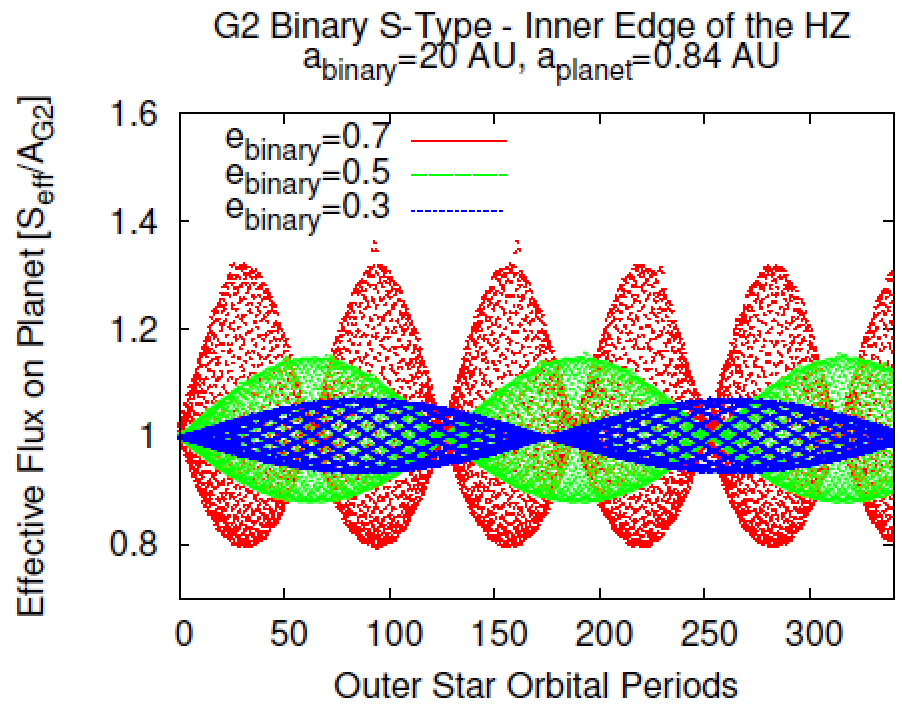


$$A \geq S_{eff} \geq B$$

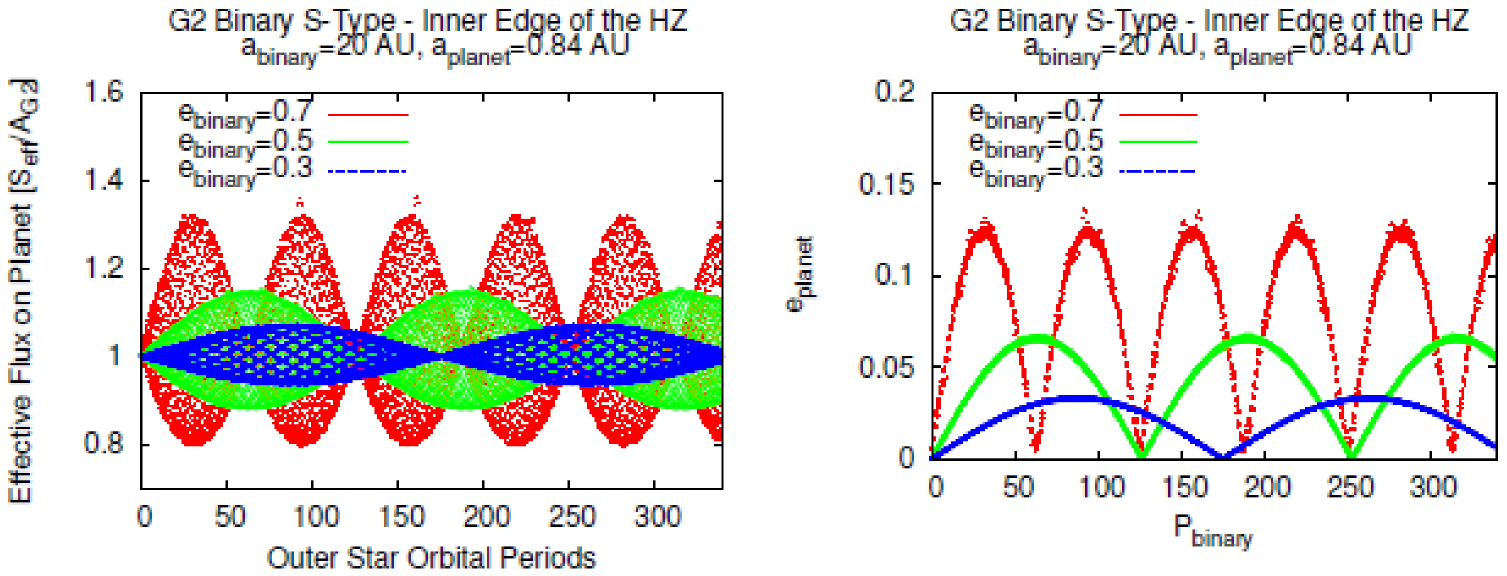
LIMITING RADIATION VALUES FOR THE INNER (A) AND OUTER (B) BORDER OF THE HZ RESPECTIVELY IN UNITS OF SOLAR CONSTANTS ($1360\,[W/m^2]$). THE VALUES WERE TAKEN FROM KASTING ET AL. (1993) ASSUMING A RUNAWAY GREENHOUSE SCENARIO FOR THE INNER LIMIT, AND A MAXIMUM GREENHOUSE EFFECT FOR THE OUTER LIMIT.

Spectral Type	A	B
F0	1.90	0.46
G2	1.41	0.36
M0	1.05	0.27

Insolation



G2 Binary S-Type - Inner Edge of the HZ
a_{binary}=20 AU, a_{planet}=0.84 AU
e_{binary}=0.7
e_{binary}=0.5
e_{binary}=0.3
Effective Flux on Planet [S_{eff}/A_{G2}]
1.6
1.4
1.2
1
0.8
0 50 100 150 200 250 300
Outer Star Orbital Periods



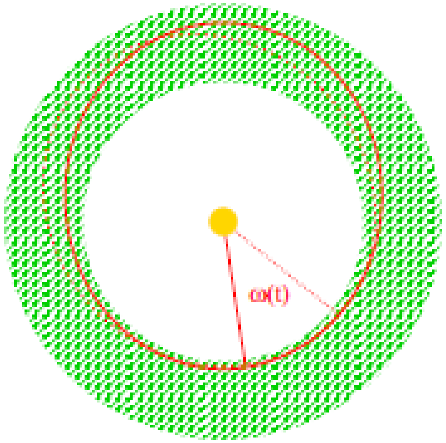
G2 Binary S-Type - Inner Edge of the HZ
a_{binary}=20 AU, a_{planet}=0.84 AU
Effective Flux on Planet [S_{eff}/A_{G2}]
Outer Star Orbital Periods
e_{binary}=0.7
e_{binary}=0.5
e_{binary}=0.3
1.6
1.4
1.2
1
0.8
0
50
100
150
200
250
300
G2 Binary S-Type - Inner Edge of the HZ
a_{binary}=20 AU, a_{planet}=0.84 AU
e_{planet}
P_{binary}
e_{binary}=0.7
e_{binary}=0.5
e_{binary}=0.3
0.2
0.15
0.1
0.05
0
0
50
100
150
200
250
300

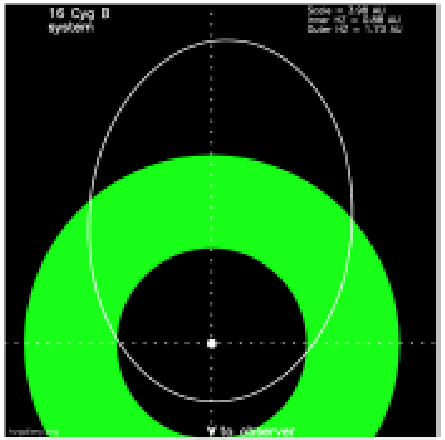
PHZ (Permanently Habitable Zone) : planet is always within habitable insolation limits ($I \leq S_{eff} \leq O$)

EHZ (Extended Habitable Zone) : planet is almost always within habitable insolation limits ($I \leq \langle S_{eff} \rangle_t \pm \sigma \leq O$)

AHZ (Average Habitable Zone) : planet is on average within habitable insolation limits ($I \leq \langle S_{eff} \rangle_t \leq O$)

Analytical estimates of PHZ



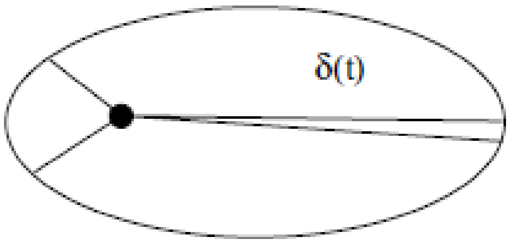


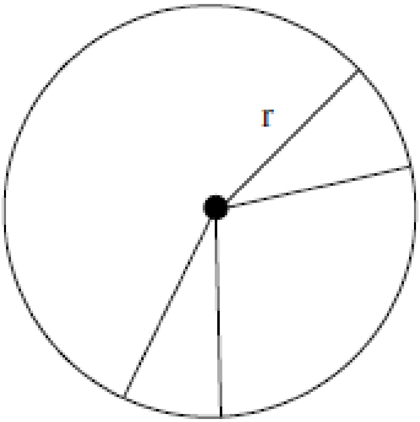
$$r_{min} = a_p(1 - e_{p,max}^2) \qquad r_{max} = a_p(1 + e_{p,max}^2) \qquad e_p^{max} = e_p^{sp} + e_p^{sec}$$

$$e_p^{sp} = \alpha \left(\frac{15}{64} \frac{\beta}{X^{5/3}} \frac{(4 + 11e_b^2)}{(1 - e_b^2)^{5/2}} + \frac{11}{4} \frac{1}{X^2} \frac{(1 + e_b)^3}{(1 - e_b^2)^3} + \frac{3}{4} \frac{1}{X^3} \frac{(1 + e_b)^4 (6 + 11e_b)}{(1 - e_b^2)^{9/2}} \right)$$

$$e_p^{sec} = e_b \beta \left(\frac{5}{4} \frac{\alpha}{X^{1/3}} \frac{3 + 2e_b^2}{(1 - e_b^2)^{1/2}} - \frac{2}{5} \gamma X^{1/3} (1 - e_b^2)^{1/2} + \frac{2}{5} X^{2/3} (1 - e_b^2) \right)^{-1}$$

Averaging Insolation Over Eccentric Orbits





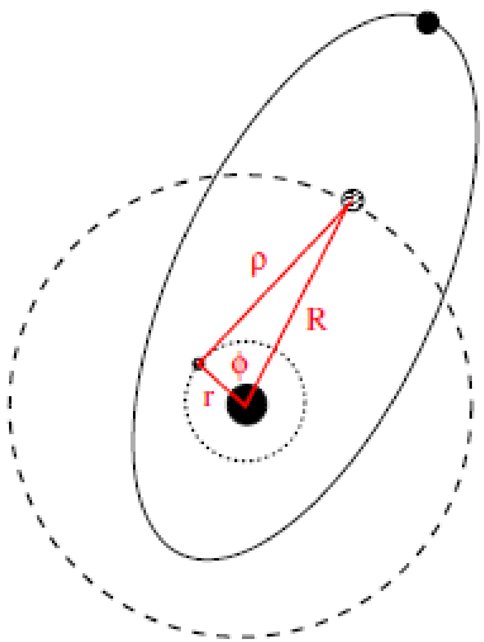
$$\langle S_1 \rangle_t = \frac{L_1}{P}\int_0^P \frac{1}{\delta^2(t)}dt \;=\; \frac{L_1 n}{2\pi}\int_0^{2\pi}\frac{1}{h}df = \frac{L_1 n}{h}$$

$$h_{ellipse} \;=\; h_{circle}$$

$$\langle S_1 \rangle_t \;=\; \frac{L_1}{r^2}$$

$r = a_p(1 - \langle e_p^2 \rangle_t)\ldots$ equivalence radius

Analytical estimates of AHZ



$$R = a_b(1 - \langle e_b^2 \rangle_t) \simeq a_b(1 - e_b^2)$$

$$\langle S_{tot} \rangle_t = \langle S_1 \rangle_t + \langle S_2 \rangle_t = \frac{L_1}{r^2} + \frac{L_2}{R^2 - r^2}$$

$$r = a_p(1 - \langle e_p^2 \rangle_t) \qquad \langle e_p^2 \rangle_t \to \langle S_1 \rangle_t$$

Georgakarakos (2003, 2005)

$$
\begin{aligned}
\overline{e_{\rm in}^2} = {} & \frac{m_3^2}{M^2}\frac{1}{X^4(1-e^2)^{9/2}}\left\{\frac{43}{8}+\frac{129}{8}e^2+\frac{129}{64}e^4+\frac{1}{(1-e^2)^{3/2}}\left(\frac{43}{8}+\frac{645}{16}e^2+\frac{1935}{64}e^4+\frac{215}{128}e^6\right)+\frac{1}{X^2(1-e^2)^3}\right. \\
& \times\left[\frac{365}{18}+\frac{44327}{144}e^2+\frac{119435}{192}e^4+\frac{256105}{1152}e^6+\frac{68335}{9216}e^8\right. \\
& \left.+\frac{1}{(1-e^2)^{3/2}}\left(\frac{365}{18}+\frac{7683}{16}e^2+\frac{28231}{16}e^4+\frac{295715}{192}e^6+\frac{2415}{8}e^8+\frac{12901}{2048}e^{10}\right)\right] \\
& +\frac{1}{X(1-e^2)^{3/2}}\left[\frac{61}{3}+\frac{305}{2}e^2+\frac{915}{8}e^4+\frac{305}{48}e^6+\frac{1}{(1-e^2)^{3/2}}\left(\frac{61}{3}+\frac{854}{3}e^2+\frac{2135}{4}e^4+\frac{2135}{12}e^6+\frac{2135}{384}e^8\right)\right] \\
& +m_*^2X^{2/3}(1-e^2)\left[\frac{225}{256}+\frac{3375}{1024}e^2+\frac{7625}{2048}e^4+\frac{29225}{8192}e^6+\frac{48425}{16384}e^8+\frac{825}{2048}e^{10}\right. \\
& \left.+\frac{1}{(1-e^2)^{3/2}}\left(\frac{225}{256}+\frac{2925}{1024}e^2+\frac{775}{256}e^4+\frac{2225}{8192}e^6+\frac{25}{512}e^8\right)\right] \\
& +m_*^2\frac{1}{X^{4/3}(1-e^2)^2}\left[\frac{8361}{4096}+\frac{125415}{8192}e^2+\frac{376245}{32768}e^4+\frac{41805}{65536}e^6\right. \\
& \left.\left.+\frac{1}{(1-e^2)^{3/2}}\left(\frac{8361}{4096}+\frac{58527}{2048}e^2+\frac{877905}{16384}e^4+\frac{292635}{16384}e^6+\frac{292635}{524288}e^8\right)\right]\right\}+2\left(\frac{C}{B-A}\right)^2.
\end{aligned}
$$

Analytical estimates of EHZ

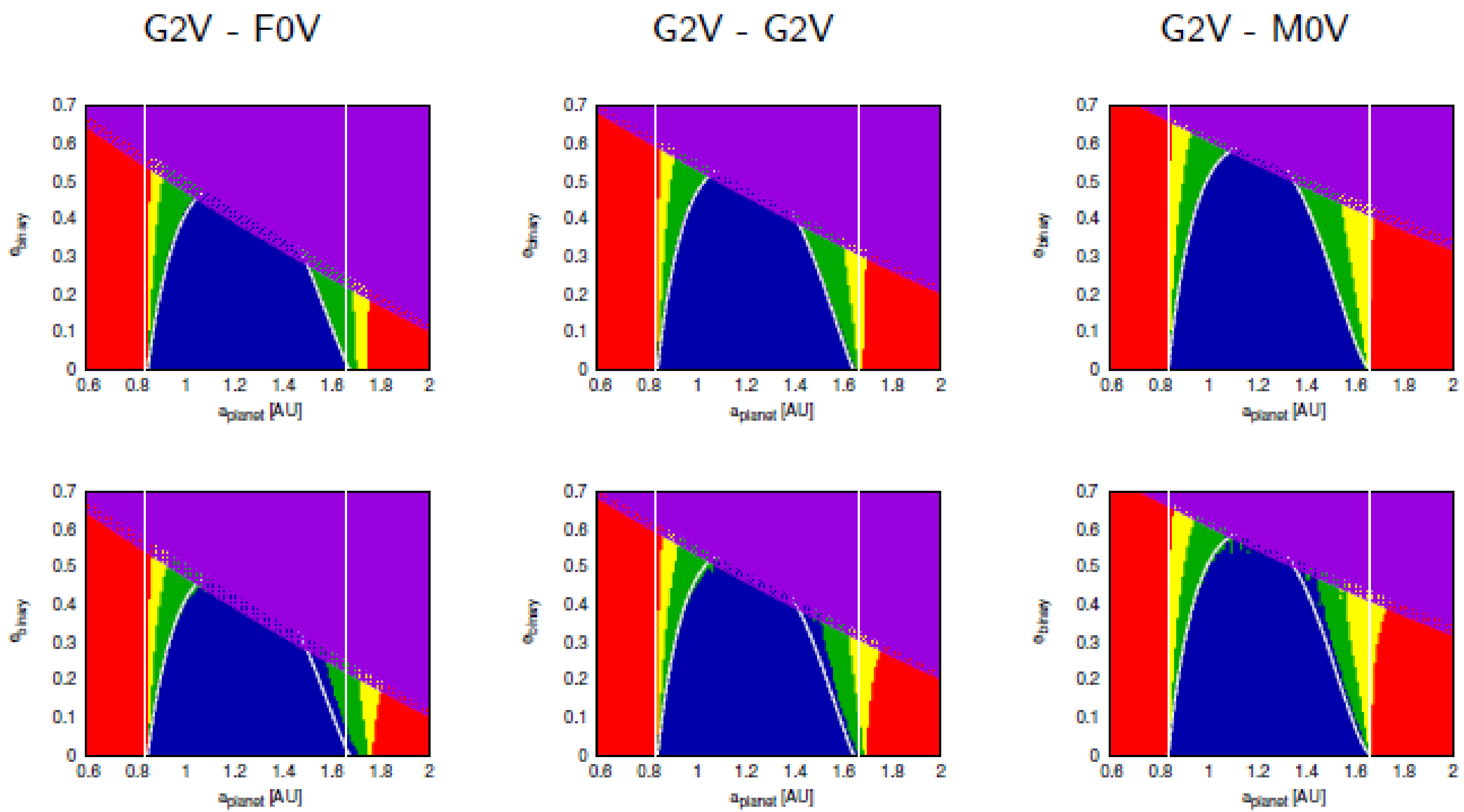
planet is almost always within habitable insolation limits:

$$I \leq \langle S_{eff} \rangle_t \pm \sigma \leq O$$

$$\sigma^2 = \langle S_{tot}^2 \rangle_t - \langle S_{tot} \rangle_t^2$$

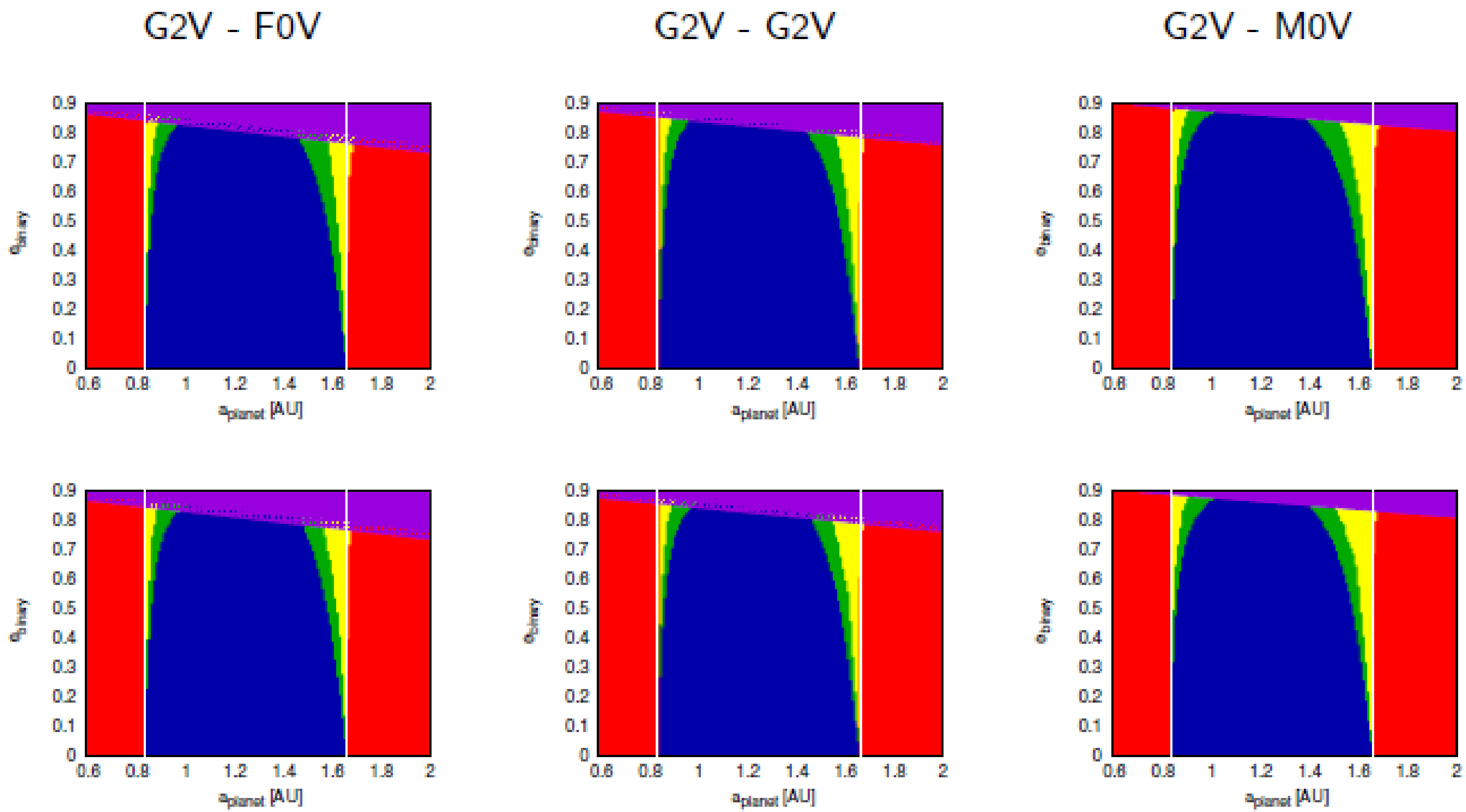
$$\begin{aligned}\sigma_X^2 \quad = \quad & \frac{L_1^2}{X_1^2 r^4} \left(-1 + 3\langle e_p^2 \rangle - 3\langle e_p^2 \rangle^2 + \langle e_p^2 \rangle^3\right) + \frac{L_1^2}{X_1^2 r^4} \sqrt{1 - \langle e_p^2 \rangle} \left(1 - \frac{\langle e_p^2 \rangle}{2} - \frac{\langle e_p^2 \rangle^2}{2}\right) \\ & - \frac{2 L_1 L_2}{X_1 X_2 (r^4 - r^2 R^2)} \left(1 - \sqrt{1 - \langle e_p^2 \rangle}\,(1 + \langle e_p^2 \rangle)\right) - \frac{2 L_2^2 r^2}{X_2^2 (r^2 - R^2)^3}\end{aligned}$$

where $X_i \in \{I_i, O_i\}$



top: analytic estimates, bottom: simulation, $a_b = 10AU$

Eggl, S., Pilat-Lohinger, E., Georgakarakos, N., Gyergyovits, M. & Funk, B.: (2012) *An Analytic Method to determine Habitable Zones for S-Type Planetary Orbits in Binary Star Systems* ApJ, in press



top: analytic estimates, bottom: simulation, $a_b = 50AU$

Eggl, S., Pilat-Lohinger, E., Georgakarakos, N., Gyergyovits, M. & Funk, B.: (2012) *An Analytic Method to determine Habitable Zones for S-Type Planetary Orbits in Binary Star Systems* ApJ, in press