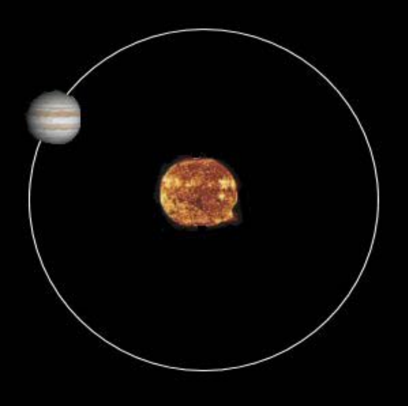
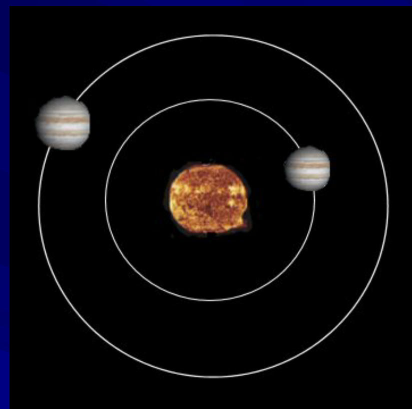
October 2012:
663 planetary systems
841 planets



- Single-Star - Single-Planet

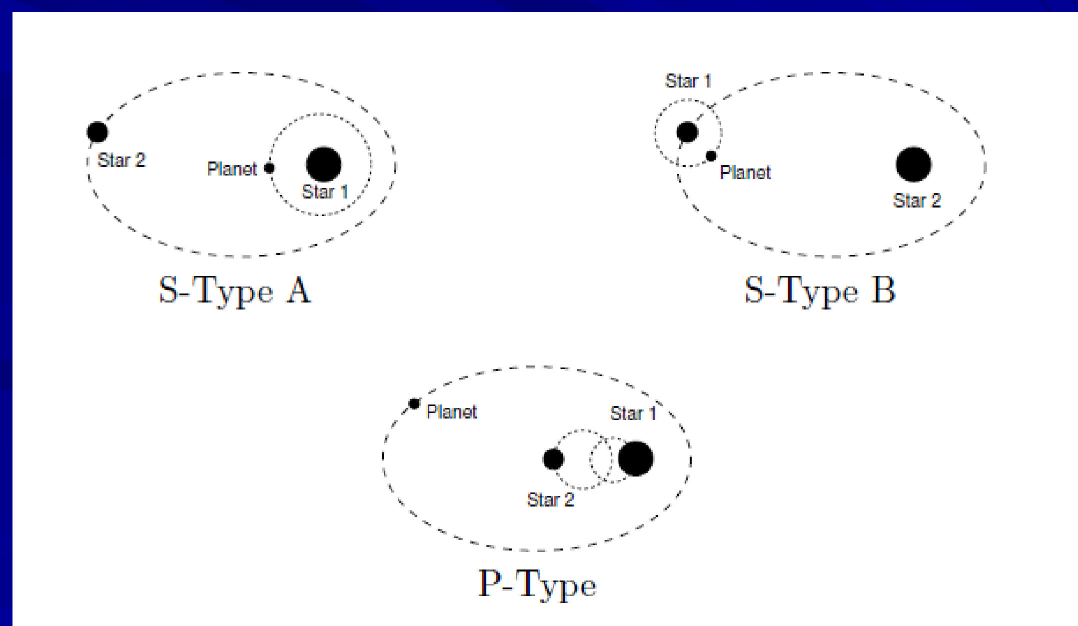


- 126 Multi-planet systems

(http://exoplanet.eu/catalog/)

57 planets
in binaries

(Roell et al., 2012, A&A)



Two Body Problem

Interaction of two masses moving under the mutual gravitational attraction described by Newton‘s universal law of gravitation:

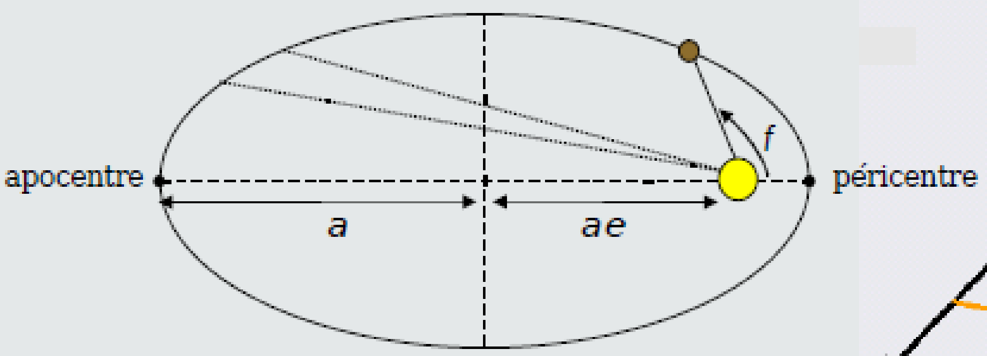
$$F = G\ (m1\ .\ m2)\ /d^2$$

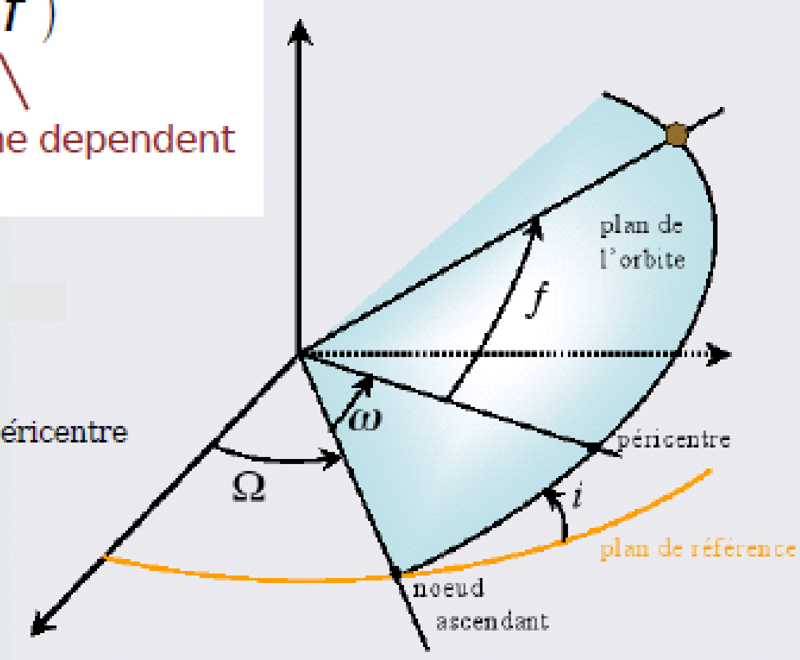
and Kepler‘s empirical laws of planetary motion

✦ Orbital elements

The planets move in ellipses with the Sun at one focus (first Kepler's law)

$$(\underbrace{a, e, i, \omega, \Omega}_{\text{constants}}, \underbrace{f}_{\text{time dependent}})$$





More than 2 bodies:

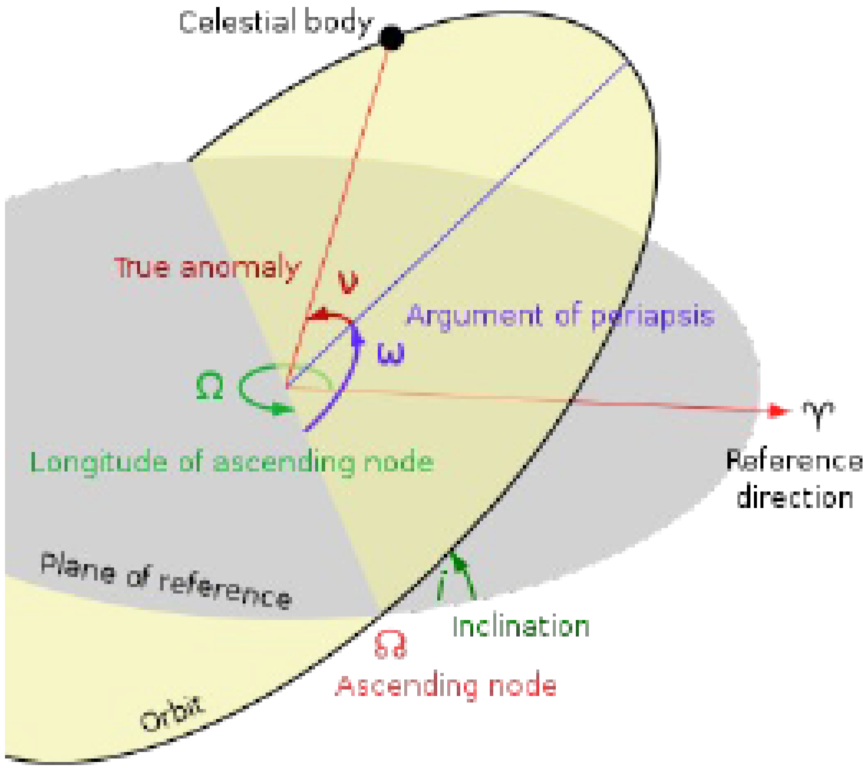
Three body problem: 18 Integrals of motion are needed
only 10 exist (Poincare)

Restricted three body problem:
2 masses + 1 masslesss body

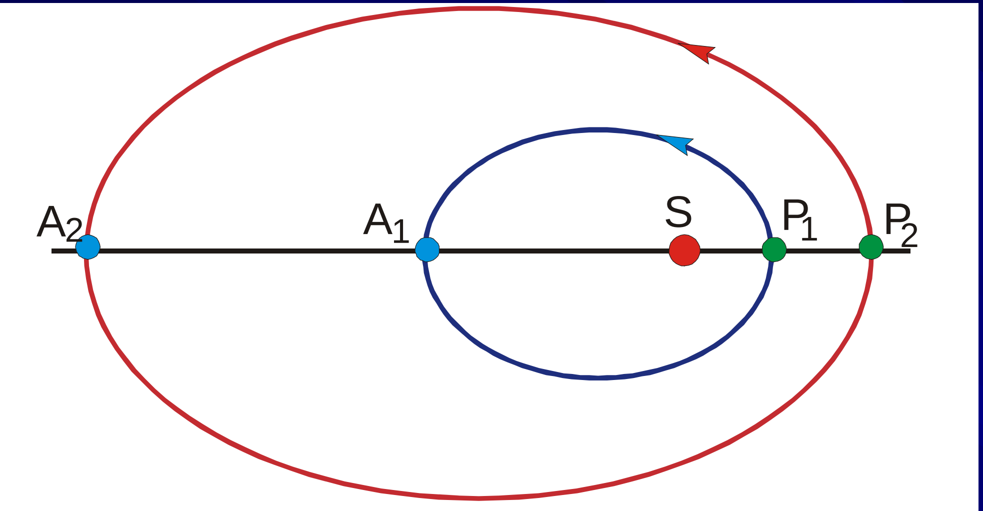
N-body problem

-→ numerical Solutions

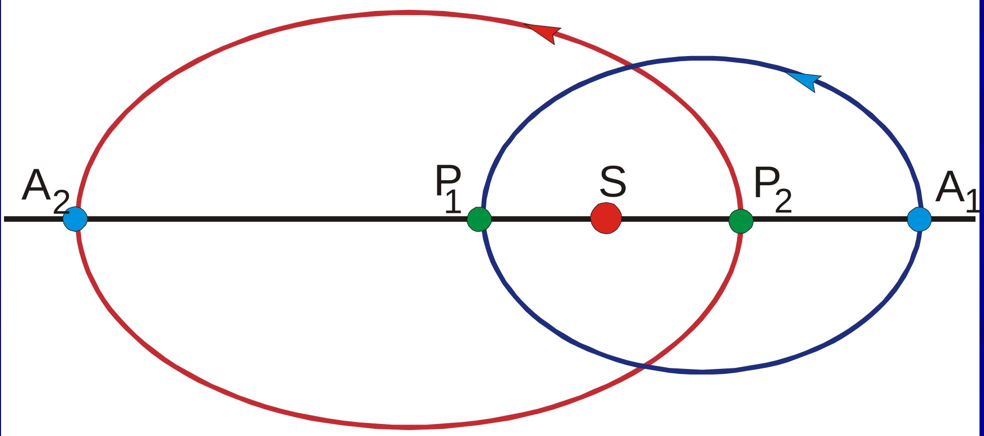
Osculating elements: $(a, e, i, \Omega, \omega, \nu)$



Resonant Motion: $n : n' = (p + q) : p$



Periastra in the same direction



Periastra in opposite directions

Periastra in the same direction

S - P_1 - P_2

S - A_1 - A_2

A_1 - S - P_2

P_1 - S - A_2

Periastra in opposite directions

S - P_1 - A_2

S - A_1- P_2

P_1 - S – P_2

A_1 - S – A_2

Equivalent in pairs, depending on the resonance

Numerical Methods

Chaos Indicators:

Fast Lyapunov Indicator (FLI)

C. Froeschle, R.Gonczi, E. Lega (1996)

MEGNO

RLI

Helicity Angle

LCE

Long-term numerical integration:

Stability-Criterion:

No close encounters within the <u>Hill‘ sphere</u>

(i)Escape time

(ii) Study of the eccentricity: maximum eccentricity

Spacing of Planets -- Hill criterion

- Convenient rough proxy for the stability of planetary systems
- In its simpliest form for planets of equal mass on circular orbits around a sun-like star

Two adjacent orbits with separation
$\Delta a_i = a_{i+1} - a_i$ have to fullfill:

$$a_{i+1} = a_i + 2kR_{Hill} \qquad (1)$$

$$k = 4 - 15$$

$$R_{Hill} = a_i(m/(3M))^{(1/3)}$$

→ mass-dependency

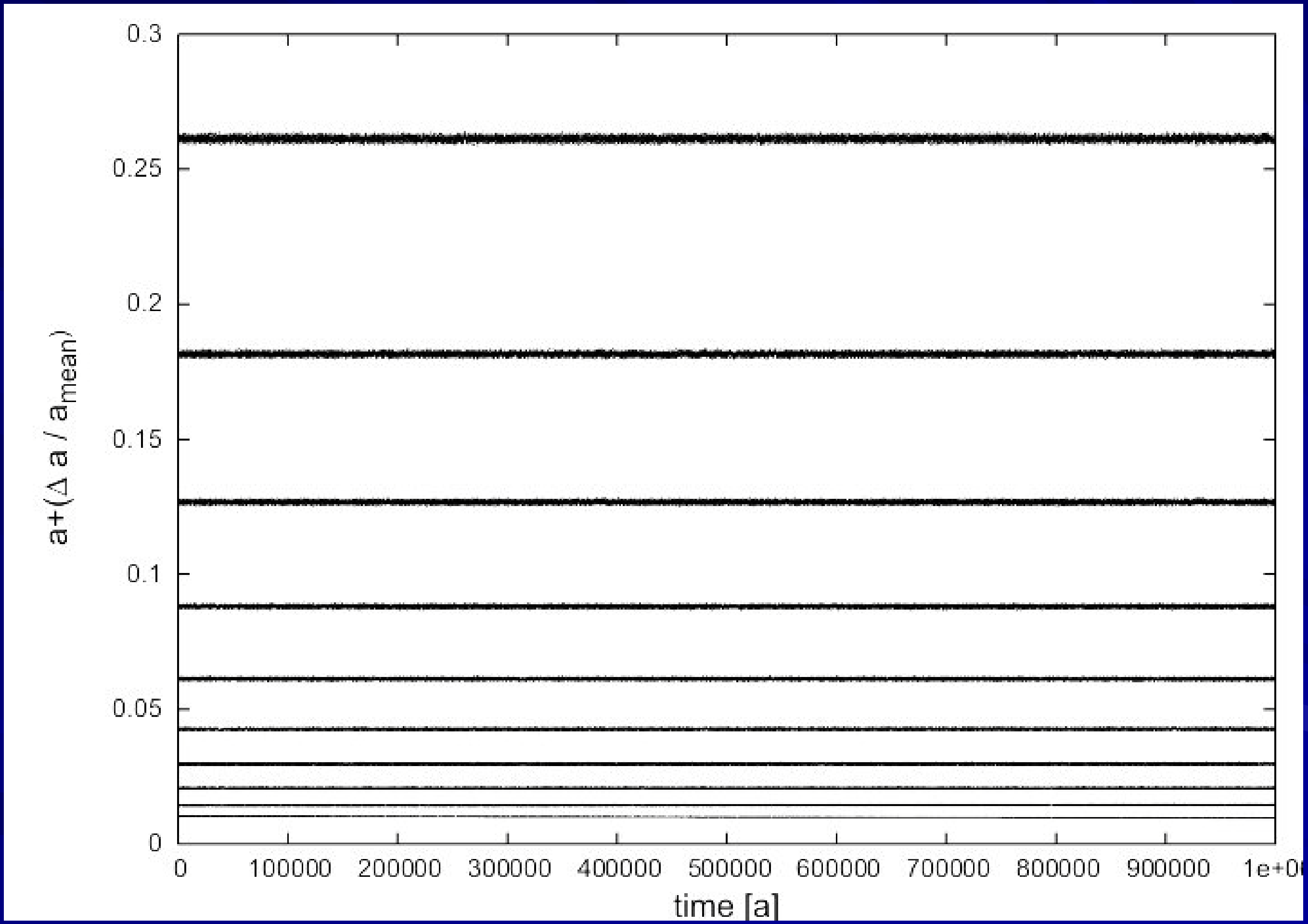
$$\frac{\Delta a_i}{a_i} \approx 2k\left(\frac{m}{3M}\right)^{1/3} \qquad (2)$$

closer spacing for smaller mass planets

Numerical Study

- Fictitious compact planetary systems:
- Sun-mass star
- up to 10 massive planets (4/17/30 Earth-masses)
- a_P= 0.01AU …… 0.26 AU
- e, incl, omega, Omega, M: 0

Compact planetary system using the Hill-Criterion: m_p=1Earth-mass



Migration of planets

Solar System: Nice Model

NICE MODEL

Alessandro Morbidelli (OCA, France)
Kleomenios Tsiganis (Thessaloniki, Greece)
Hal Levison (SwRI, USA)
Rodney Gomes (ON-Brasil)

Tsiganis et al. (2005), *Nature* 435, p. 459

Morbidelli et al. (2005), *Nature* 435, p. 462

Gomes et al. (2005), *Nature* 435, p. 466

N-body simulations:

Sun + 4 giant planets + Disc of planetesimals

- **43** simulations t~100 My:
  (e , sin$I$) ~ 0.001
  a_J=5.45 AU , $a_S=a_J 2^{2/3}$ - Δa , Δa < 0.5 AU

  U and N initially with a < 17 AU (Δa > 2 AU)
  Disc: 30-50 M_E , edge at 30-35 AU (1,000 – 5,000 bodies)

- 8 simulations for **t ~ 1 Gy** with a_S= 8.1-8.3 AU

Perturbations

Mean Motion Resonances (MMRs) or Obital Resonances

ratio of the orbital periods are integers

Secular Resonances

influence on the longitudes of perihelion or node

Initial Conditions and Computations

Planet	a (AU)	e	Inclination (deg)	ω (deg)	Ω (deg)	$\mathcal{M}$ (deg)	Mass m ($M_\odot$)
Jupiter.................	5.2028	0.0483	1.3046	275.201	100.471	183.898	0.9547907E−3
Saturn	9.5300	0.0533	2.4864	339.520	113.669	238.293	0.2858776E−3

Saturn: $a_{\_sat}$ = 8 ..... 11 AU

$m_{\_sat}$ = 1 .... 30x$m_{_Sat}$

Testplanets in the HZ:

$a_{\_tp}$ = 0.6 ..... 1.6 AU

circular motion

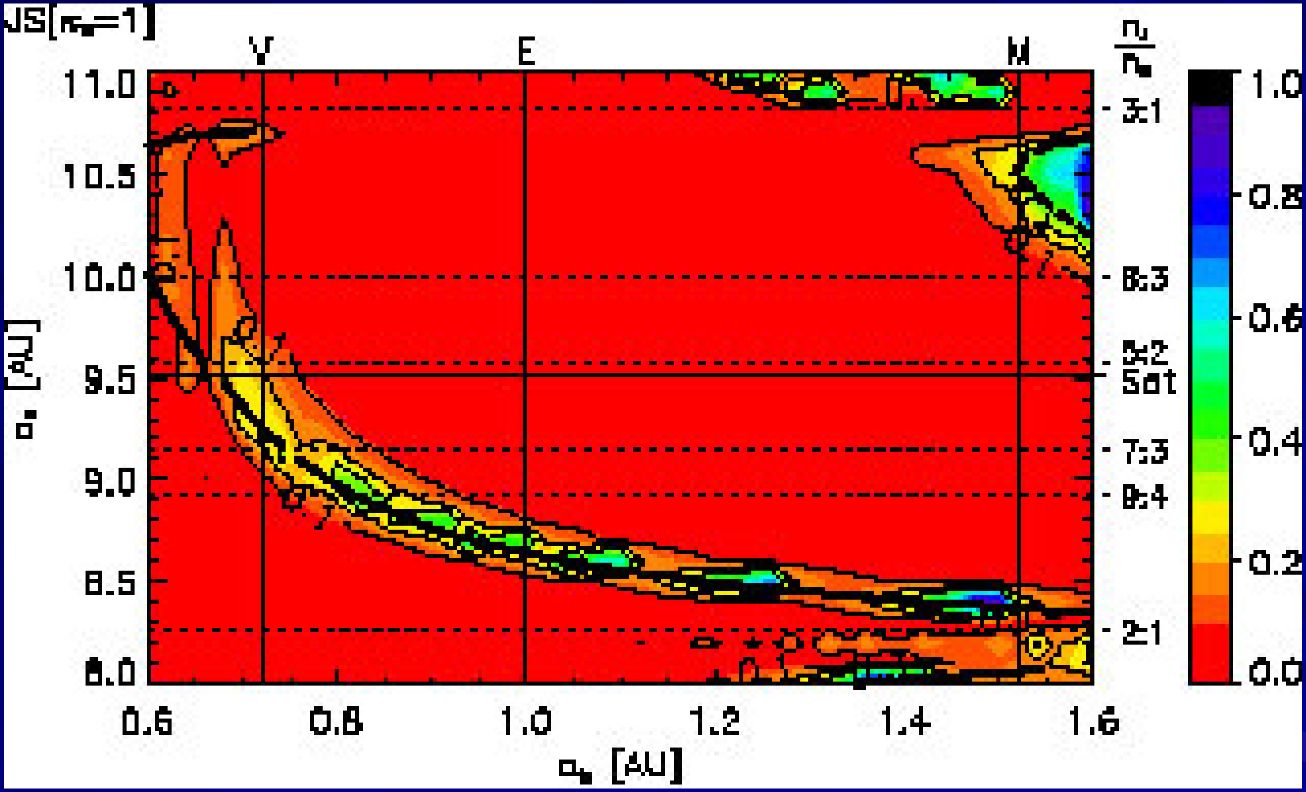
Mercury 6 (J. Chambers)

Integration time:

20 mio years

HZ: maximum ecc.

Sun – Jupiter – Saturn



(Pilat-Lohinger et al., ApJ 2008)

Secular Perturbations:

$$g = \frac{n}{4}\left[\frac{m_{\rm J}}{M_\odot}\alpha_{\rm J}^2 b_{3/2}^{(1)}(\alpha_{\rm J}) + \frac{m_{\rm S}}{M_\odot}\alpha_{\rm S}^2 b_{3/2}^{(1)}(\alpha_{\rm S})\right], \qquad (1)$$

where $\alpha_{\rm J} = a/a_{\rm J}$, $\alpha_{\rm S} = a/a_{\rm S}$ (where $a_{\rm J}$, $a_{\rm S}$, and a are the semi-major axes of Jupiter, Saturn, and the test planet, respectively), $m_{\rm J}$ and $m_{\rm S}$ are the masses of Jupiter and Saturn, $M_\odot$ is the mass of the Sun, and $b_{3/2}^{(1)}$ is a Laplace coefficient.

(See e.g. Murray & Dermott, Solar System Dynamics)