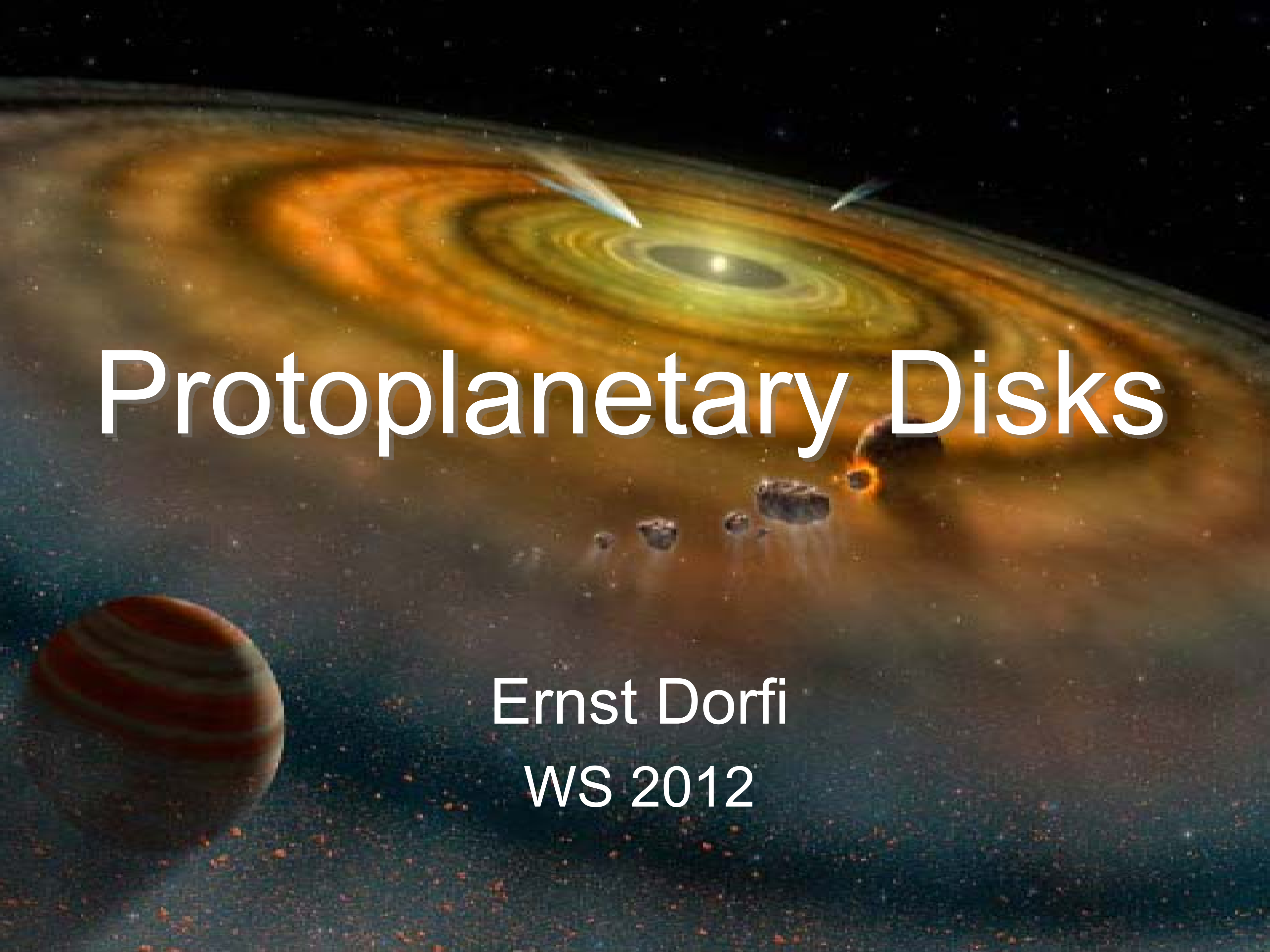
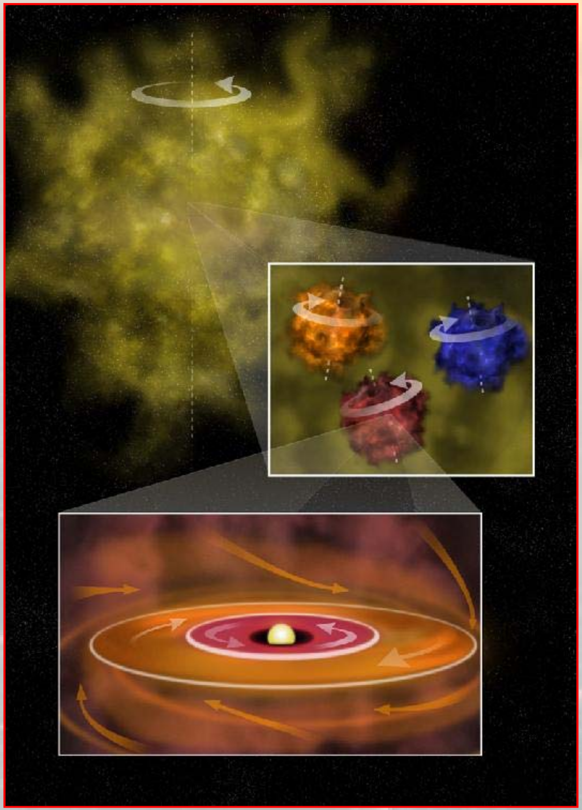


Protoplanetary Disks

Ernst Dorfi

WS 2012

Disks and star formation



Bill Saxton, NRAO/AUI/NSF

- Disks of gas and dust are observed around young stars for about 10^7 years
- Disks form because of angular momentum (conservation) , collapse from interstellar densities $n{\sim}10^5 cm^{-3}$ to stellar densities $n{\sim}10^{24} cm^{-3}$
- Interstellar clouds: Fraction of rotational energy compared to gravitational energy:

$$\beta = \frac{E_{\mathrm{rot}}}{|E_{\mathrm{grav}}|}$$

$$J_{\mathrm{cloud\,core}} = 10^{54} \mathrm{g\,cm^2 s^{-1}}$$

- $M=1M_\odot$ and $R=0.05pc$ leads typically to

$$j_{\mathrm{cc}} = \frac{J_{\mathrm{cloud\,core}}}{1\,M_\odot} = 10^{20} \mathrm{cm^2 s^{-1}}$$

$$j_{\mathrm{cc}} = \sqrt{G M_* r_{\mathrm{eq}}}$$

- Uniformly rotating cloud with same β settles to centrifugal equilibrium at r_{eq}~100 AU

Some Solar system values

NASA/Cassini**: Jupiter** with shadow of Europa

- Masses of planets are only 0.13% compared to the Solar mass $M_\odot$ = 1.989 x10^{33} g
- Angular momentum of our sun:

$$J_\odot = k^2 M_\odot R_\odot^2 \Omega \simeq 3 \cdot 10^{48}\,\mathrm{g\,cm^2 s^{-1}}$$

$$k^2 \simeq 0.1$$

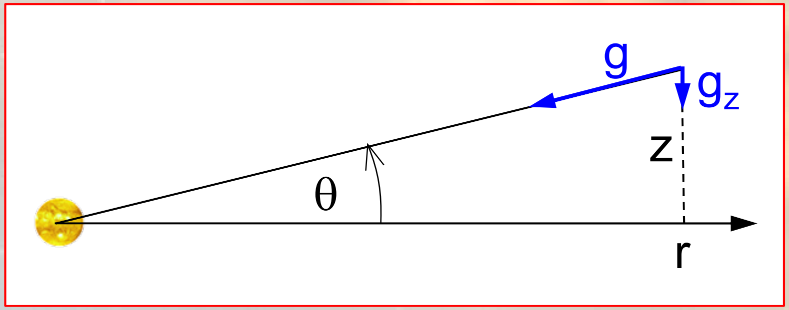
$R_\odot$ = 6.96 x10^{10} cm, Ω=2.9x10^{-6}s^{-1} (corresponding to 25 days rotation period), k typical value for main sequence star

- Orbital angular momentum of Jupiter:

$$J_{\mathrm{Jup}} = M_{\mathrm{Jup}} \sqrt{G M_\odot a_{\mathrm{Jup}}} = 2 \cdot 10^{50}\,\mathrm{g\,cm^2 s^{-1}}$$

- Small values compared to interstellar clouds, substantial segregation of mass and angular momentum necessary

Vertical hydrostatic structure (I)



$$\frac{dP}{dz} = -\rho\, g_z$$

$$\frac{1}{\rho}\frac{d\rho}{dz} = -\frac{G\,M_*\;z}{c_s^2\,(r^2+z^2)^{3/2}}$$

- Hydrostatic equilibrium perpendicular to the disc plane, i.e. $v_z = 0$
- Gravity only due to central object with mass M_*, ignoring the disk mass

$$g_z = g\sin\theta = \frac{G\,M_*}{(r^2+z^2)}\frac{z}{(r^2+z^2)^{1/2}}$$

- Isothermal equation of state, where $c_s = c_s(T)$ constant sound velocity

$$P = c_s^2 \rho$$

- Vertical density structure by simple integration, ρ_0 equatorial density

$$\rho = \rho_0 \exp\left[\frac{GM_*}{c_s^2}\left(\frac{1}{(r^2+z^2)^{1/2}} - \frac{1}{r}\right)\right]$$

Vertical hydrostatic structure (II)

$$z \ll r:$$

$$\frac{1}{\rho}\frac{d\rho}{dz} = -\frac{G\,M_*\,z}{c_s^2 r^3} = -\frac{\Omega^2}{c_s^2} z$$

$$h = \frac{c_s}{\Omega}$$

$$\rho = \rho_0\, e^{-z^2/2h^2}$$

- Thin disc assumption simplifies hydrostatic structure
- Radial balance from Keplerian motion v_K, Ω Keplerian angular velocity

$$\frac{GM_*}{r^2} = \frac{v_K^2}{r} = \frac{\Omega^2 r^2}{r}$$

- Vertical scale height h by ratio between sound velocity and Keplerian angular velocity leads to simpler density structure
- Azimutal Mach number M from velocity ratio

$$\frac{h}{r} = \frac{c_s}{r\Omega} = \mathcal{M}^{-1} \qquad \mathcal{M} = \frac{v_K}{c_s}$$

Surface density

- Simplification by integration of the gas density over the vertical direction, leads to surface density Σ

$$\Sigma(r,\varphi) = \int_{-\infty}^{\infty} \rho(r,\varphi,z)\, dz$$

- Vertical hydrostatic equilibrium assumed:

$$\Sigma = \rho_0 \int_{-\infty}^{\infty} e^{-z^2/2h^2}\, dz$$

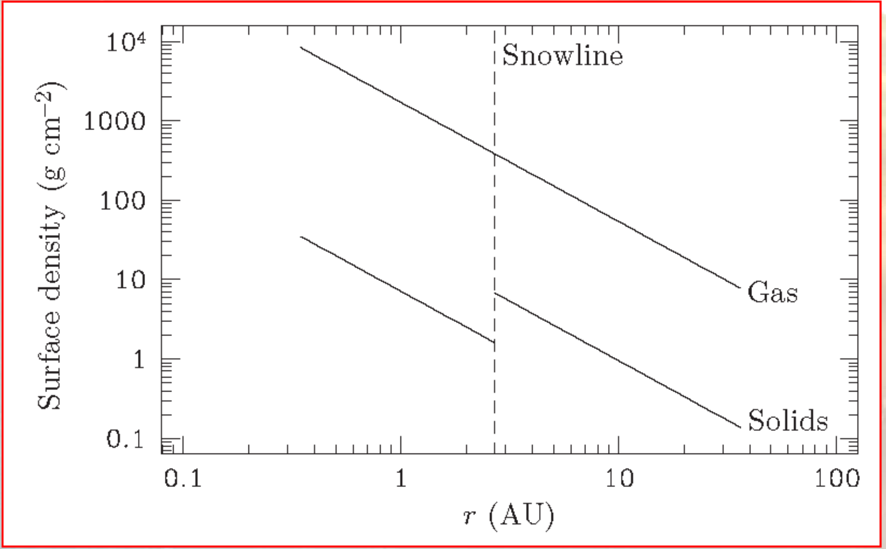
Remember: $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$

- Simple relation between central density ρ_0 and surface density Σ, but quantities depend on radius, e.g. h=h(r)

$$\rho_0 = \frac{1}{\sqrt{2\pi}} \frac{\Sigma}{h}$$

- Example: T=100 K (r =1 AU) yields with μ=2.6 and a central mass of $1M_\odot$: $c_s^2 = \frac{k_B T}{\mu m_p}$ $\Rightarrow$ $c_s \simeq 0.6\,\mathrm{km/s}$ $\frac{h}{r} \simeq 0.02$

Minimum mass Solar nebula



- Hayashi (1981): lower limit of surface density, spread each planetary mass across its orbit:

$$\Sigma(r) = 1.7 \cdot 10^3 \left(\frac{r}{[1\,\mathrm{AU}]}\right)^{-3/2} \mathrm{g\,cm^2}$$

- Integration up to 30 AU yields to 0.01 $M_\odot$, comparable to other disc observations
- Surface density of solids, presence of icy particles in outer disk

$$\Sigma_{\mathrm{rock}}(r) = 7.1 \cdot 10^3 \left(\frac{r}{[1\,\mathrm{AU}]}\right)^{-3/2} \mathrm{g\,cm^2} \quad \mathrm{for} \quad r < 2.7\,\mathrm{AU}$$

$$\Sigma_{\mathrm{rock+ice}}(r) = 30 \cdot 10^3 \left(\frac{r}{[1\,\mathrm{AU}]}\right)^{-3/2} \mathrm{g\,cm^2} \quad \mathrm{for} \quad r > 2.7\,\mathrm{AU}$$

Hydrodynamic Equations

- Cylindrical coordinates (r, φ, z) and $\vec{u} = (u_r, u_\varphi, u_z)$
- Equation of continuity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad \Rightarrow \quad \frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial\, r \rho u_r}{\partial r} + \frac{1}{r}\frac{\partial \rho u_\varphi}{\partial \varphi} + \frac{\partial \rho u_z}{\partial z} = 0$$

- Navier-Stokes equation for a viscous fluid with kinematic viscosity ν and bulk (=second) viscosity ζ

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\,\vec{u} = -\frac{1}{\rho}\nabla P - \nabla \Phi + \nu \Delta \vec{u} + \left(\frac{\nu}{3} + \zeta\right) \nabla\left(\nabla \cdot \vec{u}\right)$$

$$\begin{aligned}
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_r}{\partial \varphi} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\varphi^2}{r} &= -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r} \\
\frac{\partial u_\varphi}{\partial t} + u_r \frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_\varphi}{\partial \varphi} + u_z \frac{\partial u_\varphi}{\partial z} - \frac{u_r u_\varphi}{r} &= -\frac{1}{\rho r}\frac{\partial P}{\partial \varphi} - \frac{1}{r}\frac{\partial \Phi}{\partial \varphi} \\
\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_z}{\partial \varphi} + u_z \frac{\partial u_z}{\partial z} &= -\frac{1}{\rho}\frac{\partial P}{\partial z} - \frac{\partial \Phi}{\partial z}
\end{aligned}$$

Radial forces

- Stationary solutions, i.e. u_r=0

$$\cancel{\frac{\partial u_r}{\partial t}} + u_r \cancel{\frac{\partial u_r}{\partial r}} + \frac{u_\varphi}{r} \cancel{\frac{\partial u_r}{\partial \varphi}} + u_z \cancel{\frac{\partial u_r}{\partial z}} - \frac{u_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{\partial \Phi}{\partial r}$$

$$\Rightarrow \quad \frac{u_\varphi^2}{r} = \frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{GM_*}{r^2}$$

- Pressure P_0 at r_0 varies locally like $P(r) = P_0 \left(\frac{r}{r_0}\right)^{-n}$ with $P_0 = c_s^2 \rho_0$

- Pressure is decreasing outwards, hence u_φ is always smaller than Keplerian velocity v_K, e,g. n=3, h/r=0.05, $\Sigma \sim r^{-1}$

$$u_\varphi = v_K \left(1 - n\frac{c_s^2}{v_K^2}\right)^{1/2} \quad \Rightarrow \quad u_\varphi \simeq 0.996\, v_K$$

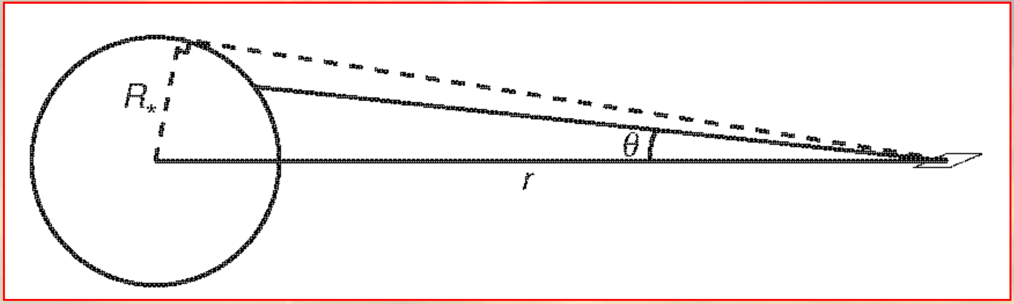
Simple temperature structure

- Disk temperature reaches equilibrium on time scales shorter than stellar evolution time scales
- Temperature: Balance between heating (viscous dissipation, irradiation, accretion) and cooling (radiation, evaporation)
- Accreted material related to the stellar luminosity absorbed by the disk (~1/4), i.e. the accretion rate can be estimated for R= 2 $R_\odot$ and L = 1 $L_\odot$

$$\frac{GM_*\dot{M}}{R_*} \simeq \frac{1}{4}L_* \quad \Rightarrow \quad \dot{M} \simeq 2 \cdot 10^{-8} M_\odot \, \mathrm{yr}^{-1}$$

- Temperature structure depends on disk geometry, can be flat, warped or flared

Razor-thin disk (I)



- Flux F though a surface in the equatorial plane, assuming a constant Intensity at the stellar surface I_*

$$F = \int I_* \sin\theta \cos\phi \, d\Omega$$

$$-\pi/2 \leq \phi \leq \pi/2 \qquad 0 < \theta < \sin^{-1}\left(\frac{R_*}{r}\right)$$

$$d\Omega = \sin\theta \, d\theta d\phi$$

$$F = I_* \int_{-\pi/2}^{\pi/2} \cos\phi \mathrm{d}\phi \int_0^{\sin^{-1}(R_*/r)} \sin^2\theta \mathrm{d}\theta$$

Razor-thin disk (II)

- Integration leads to

$$F = I_* \left[\sin^{-1}\left(\frac{R_*}{r}\right) - \left(\frac{R_*}{r}\right)\sqrt{1-\left(\frac{R_*}{r}\right)^2} \right]$$

- Stellar surface I_* related to stellar temperature: $I_* = \frac{\sigma}{\pi} T_*^4$
- Disk temperature

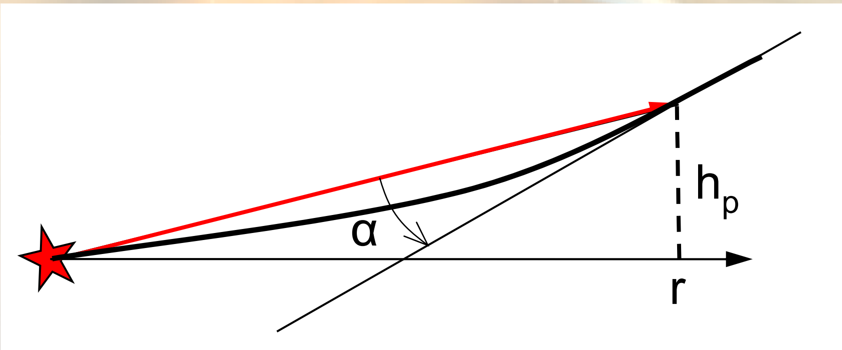
$$\left(\frac{T_{\rm disk}}{T_*}\right)^4 = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{R_*}{r}\right) - \left(\frac{R_*}{r}\right)\sqrt{1-\left(\frac{R_*}{r}\right)^2} \right]$$

- Integration over the whole disk at both sides leads

$$L_{\rm disk} = 2 \times \int_{R_*}^{\infty} 2\pi r \sigma T_{\rm disk}^4 {\rm d}r = \frac{1}{4} L_*$$

- Taylor expansion for large radii → $T_{\rm disk}(r) \propto r^{-3/4}$

Flared disks (I)



$$\alpha = \frac{dh_p}{dr} - \frac{h_p}{r}$$

- Absorption height is function of distance $h_p(r)$, (details in Kenyon & Hartmann 1987),
- Increased absorption of stellar radiation produces larger IR-excess
- Simplified for a central point source and for $r \gg R_*$
- Absorption height depends also on the opacity, i.e. $h_p(r) \neq h(r)$
- Equilibrium: Absorption equals emission (per unit disk area)

$$Q_+ = 2\alpha \left(\frac{L_*}{4\pi r^2} \right) = Q_- = 2\sigma T_{\rm disk}^4$$

Flared disks (II)

- Disk temperature profile

$$T_{\text{disk}} = \left(\frac{L_*}{4\pi\sigma}\right)^{1/4} \alpha^{-1/4} r^{-1/2}$$

- Central star radiates as a black body

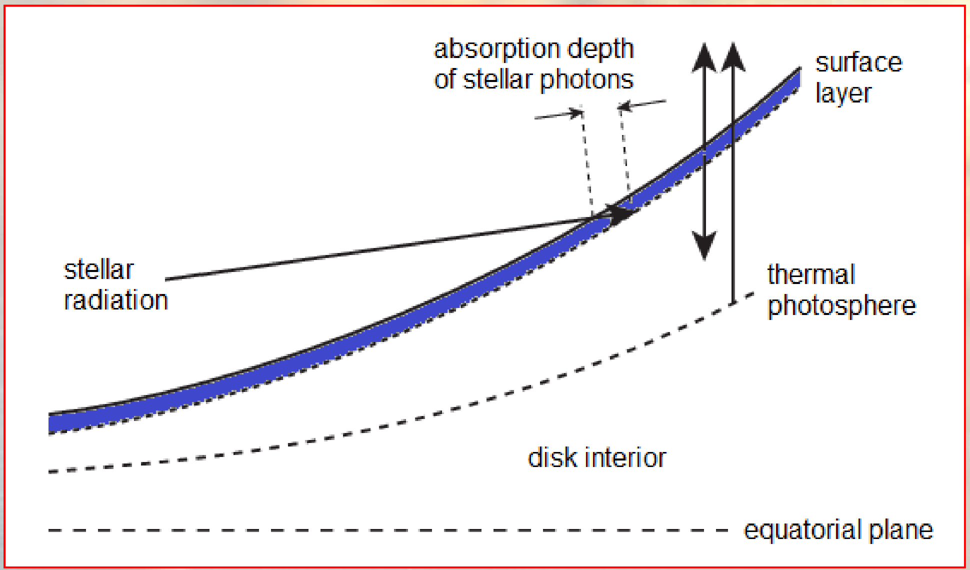
$$L_* = 4\pi\sigma R_* T_*^4$$

$$\frac{T_{\text{disk}}}{T_*} = \left(\frac{R_*}{r}\right)^{1/2} \alpha^{-1/4}$$

- For optically thick disks we can assume a constant ratio between h_p and h, i.e. $h_p \sim h$ and solve equations for α
- Temperature profile for large radii in a flared disk (Kenyon and Hartmann 1987):

$$\Rightarrow \quad T_{\text{disk}}(r) \propto r^{-1/2}$$

Radiation within disks



$$\epsilon = 1, \quad \lambda \leq 2\pi a$$
$$\epsilon = \frac{2\pi a}{\lambda}, \quad \lambda > 2\pi a$$

$$\epsilon \sim \lambda^{-1}$$

- Dust particles are most important opacity source
- Absorption of stellar radiation (around 1 μm), reemission at longer wavelengths in IR
- About half of the incoming flux is radiated into space, half is heating the disk
- Radiating properties of the dust particles determine temperature
- Opacity from Mie-theory, particles with radius a

Dust Temperature

$$Q_{\mathrm{cool}} = 4\pi a^2 \epsilon_{\mathrm{d}} \sigma T_{\mathrm{d}}^4$$

$$Q_{\mathrm{heat}} = \pi a^2 \epsilon_* F_*$$

$$L_* = 4\pi R_*^2 \sigma T_*^4 \qquad Q_{\mathrm{heat}} = Q_{\mathrm{cool}}$$

$$\pi a^2 \epsilon_* \frac{4\pi R_*^2 \sigma T_*^4}{4\pi r^2} = 4\pi a^2 \epsilon_{\mathrm{d}} \sigma T_{\mathrm{d}}^4$$

$$\frac{T_{\mathrm{d}}}{T_*} = \left(\frac{\epsilon_*}{\epsilon_{\mathrm{d}}}\right)^{1/4} \left(\frac{R_*}{2r}\right)^{1/2}$$

$$\epsilon \propto \lambda^{-1} \propto T \qquad \frac{\epsilon_*}{\epsilon_{\mathrm{d}}} = \frac{T_*}{T_{\mathrm{d}}}$$

$$\frac{T_{\mathrm{d}}}{T_*} = \left(\frac{R_*}{2r}\right)^{2/5}$$

- Simple calculation of the dust temperature T_d through equilibrium between heating (absorption) Q_{heat} and cooling (emission) Q_{cool}
- Amount of absorbed stellar radiation ε_*
- Dust emission coefficient ε_d
- Emission scaling according to black body, Wien's law
- Results in simple power-law, e.g.

$$T_* = 4000\,\mathrm{K} \quad R_* = 2\,R_\odot$$

$$\rightarrow \quad T_{\mathrm{d}} = 470 \left(\frac{r}{[1\,\mathrm{AU}]}\right)^{-2/5} \mathrm{K}$$

Condensation of dust

$$G = H - TS$$

$$G = G(\mu_i)$$

Species	Composition	T_{cond} [K]
Methane	CH_4	41
Argon hydrate	$Ar.6H_2O$	48
Methane hydrate	$CH_4.7H_2O$	78
Ammonia hydrate	$NH_3.H_2O$	131
Water ice	H_2O	182
Magnetite	Fe_3O_4	371
Troilite	FeS	704
Forsterite	Mg_2SiO_4	1354
Perovskite	$CaTiO_3$	1441
Aluminum oxide	Al_2O_3	1677

Lodders (2003) for P=10^{-4} bar

- Chemical composition of the disk difficult to calculate, chemical elements are distributed in different molecules, e.g. Oxygen in CO, H_2O, Fe_3O_4, Mg_2SiO_4
- Simplification: Thermodynamical equilibrium, minimize the Gibbs free energy G, system consists of several phases, μ_i chemical potential
- E.g.: Lodders (2003) includes 2000 gaseous species and 1600 different condensates
- Condensation depends basically on temperature

Ionization of disks

$$T \sim \frac{\chi}{10\, k_\mathrm{B}}$$

- Temperatures range in disks from a few 1000 K at the vicinity of the star to few 10 K at outer boundary
- Disks are mainly neutral because even χ=4.34eV for Potassium requires T ≥ 4000 K
- Small amounts of charged particles responsible for coupling to magnetic fields
- Sources of ionization:
  - Thermal ionization (Saha equation)

$$\frac{n^+ n_\mathrm{e}}{n} = \frac{2U^+}{U} \left(\frac{2\pi m_\mathrm{e} k_\mathrm{B} T}{h^2}\right)^{2/3} \exp\left(-\frac{\chi}{k_\mathrm{B} T}\right)$$

  - Radioactive decay
  - Energetic particles and photons (cosmic rays and X-ray flares):

$$\sigma(E) = 8.5 \cdot 10^{-23} \left(\frac{E}{[\mathrm{keV}]}\right)^{-2.81} \mathrm{cm}^2$$

Surface density

$$\Sigma(r,\varphi) = \int_{-\infty}^{\infty} \rho(r,\varphi,z)\,dz$$

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{r}\frac{\partial\, r\Sigma u_r}{\partial r} = 0$$

$$\Omega \sim r^{-3/2}$$

- The equation of continuity in axial symmetry after integration over the vertical direction
- Conservation of angular momentum with viscous transport due to torques

$$r\frac{\partial(r^2\Omega\Sigma)}{\partial t} + \frac{\partial(r^2\Omega \cdot r\Sigma u_r)}{\partial r} = \frac{1}{2\pi}\frac{\partial G}{\partial r}$$

$$G = 2\pi r \cdot \nu\Sigma r\frac{\partial\Omega}{\partial r} \cdot r = 2\pi\nu\Sigma r^3\frac{\partial\Omega}{\partial r}$$

- Viscous evolution surface density (ν kinematic viscosity)

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial(\nu\Sigma r^{1/2})}{\partial r}\right]$$

Viscosity time scales

$$X = 2r^{1/2}$$

$$f = \frac{3}{2}\Sigma X$$

$$D = \frac{12\nu}{X^2}$$

- Diffusive nature can be seen by introducing new variables
- Evolution of the surface density given by a diffusion equation with diffusion coefficient D

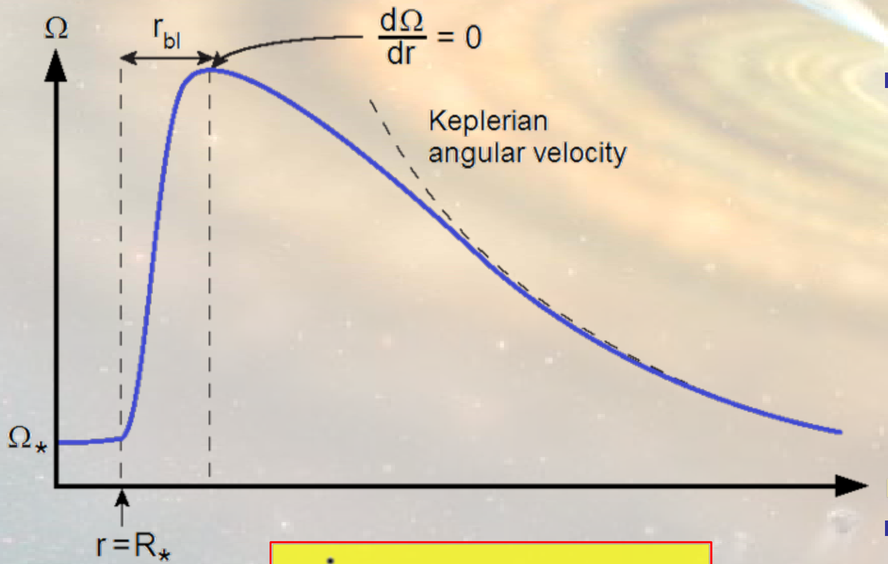
$$\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial X^2}$$

$$\tau_{\rm diff} = \frac{\Delta X^2}{D}$$

$$\tau_\nu \simeq \frac{r^2}{\nu}$$

- Viscous time scale over distance ΔX
- Physical time scale for viscous disk evolution
- τ_ν of the order of 10^6 years for protostellar disks with ~ $1M_\odot$

Solutions to the disk equation (I)



- Stationary solution with viscosity allows simple integration

$$\frac{d(r\Sigma u_r \cdot r^2\Omega)}{dr} = \frac{1}{2\pi}\frac{d}{dr}\left(2\pi\nu\Sigma r^3\frac{d\Omega}{dr}\right)$$

$$r\Sigma u_r \cdot r^2\Omega = \nu\Sigma r^3\frac{d\Omega}{dr} + C$$

- Mass accretion rate

$$-\dot{M}r^2\Omega = 2\pi\nu\Sigma r^3\frac{d\Omega}{dr} + C$$

- Boundary layer couples Keplerian disk to stellar rotation with Ω_*
- Viscous stress vanishes at $R_* + r_{bl}$

$$\dot{M} = -2\pi r\Sigma u_r$$

$$\frac{d\Omega}{dr} = 0$$

$$C = -\dot{M}(R_* + r_{\rm bl})^2\Omega_{\rm max}$$

Solutions to the disk equation (II)

- Boundary layer is small compared to stellar radius, i.e.

$$R_* + r_{\mathrm{bl}} \simeq R_*$$

- Angular velocity at boundary layer approximated by Keplerian

$$\Omega_{\max} \simeq \sqrt{\frac{GM_*}{R_*^3}} \quad \rightarrow \quad C \simeq -\dot{M} R_*^2 \sqrt{\frac{GM_*}{R_*^3}}$$

- For a Keplerian disk ($\Omega \sim r^{-3/2}$) we get a steady state solution of the disk structure with *a zero torque boundary* at the inner edge

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left(1 - \sqrt{\frac{R_*}{r}}\right) \quad \rightarrow \quad \Sigma \sim \nu^{-1}$$

- Surface density scales for large radii with the kinematic viscosity
- **BUT:** Disk can be truncated earlier by magnetic fields (e.g. at T Tauri stars) , complex physics at the inner boundary

Temperature in accreting disks

- Dissipation of energy from viscous friction is given by

$$D(r) = \frac{G}{4\pi r}\frac{d\Omega}{dr} = \nu\Sigma r^2\left(\frac{d\Omega}{dr}\right)^2 = \frac{9}{8}\nu\Sigma\Omega^2$$

where last term is calculated in case of Keplerian velocities

- The energy is radiated away by a black body

$$D(r) = \sigma T_{\rm disk}^4 \quad \Rightarrow \quad T_{\rm disk}^4 = \frac{3GM_*\dot{M}}{8\pi\sigma r^3}\left(1 - \sqrt{\frac{R_*}{r}}\right)$$

- Temperature of viscous disks is independent of the kinematic viscosity for a steady disk and for large radii $r \gg R_*$

$$T \sim r^{-3/4}$$

- Accretion rate of $10^{-7} M_\odot$/yr , $M \sim 1 M_\odot$ leads at 1AU: $T_{disk} \sim 150K$

Vertical disk structure

$$\frac{dP}{dz} = -\rho g_z$$

$$F_z \gg F_r$$

$$\frac{dF_z}{dz} = \frac{9}{4}\rho\nu\Omega^2$$

$$\frac{dT}{dz} = -\frac{3\kappa_R\rho}{16\sigma T^3}F_z$$

- Dissipation of gravitational energy closer to mid-plane
- Vertical temperature gradient will exist, transport of energy by radiation or turbulence
- Hydrostatic equilibrium in vertical direction
- Energy generation by viscous dissipation
- Radiative transport in optical thick media, κ_R is the Rosseland-mean opacity, importance of dust particles
- Equation of state, e.g. adiabatic with γ=7/5
- Gradients to steep, transport by convection leads to vertical adiabatic gradient

→ Detailed structure by numerical solutions

Mid-plane temperature

- Energy dissipation assumed to located at z=0

$$F_z(0) = \sigma T_{\rm disk}^4 \simeq F_z(z)$$

- Define optical depth τ to the disk mid-plane

$$\tau = \frac{1}{2}\kappa_{\rm R}\Sigma$$

$$-\frac{16\sigma}{3\kappa_{\rm R}}\int_0^z T^3 dT = \sigma T_{\rm disk}^4 \int_0^z \rho(z')dz'$$

$$-\frac{4\sigma}{3\kappa_{\rm R}} T^4 \Big|_{T_z}^{T_{\rm disk}} = \sigma T_{\rm disk}^4 \int_0^z \rho(z')dz' = \sigma T_{\rm disk}^4 \frac{\Sigma}{2}$$

$$T_c \gg T_{\rm disk}$$

- Viscous disks will be substantially hotter in the mid-plane

$$\left(\frac{\mathrm{T_z}}{\mathrm{T_{disk}}}\right)^4 \simeq \frac{3}{4}\tau$$

- For τ=100: $T_c \approx 3T_{disk}$, important for condensation, ice on grains, chemistry, molecules, ...

Angular momentum transport

- Disk temperature independent on viscosity but density structure as well as time scale of evolution are determined
- Physical understanding and description of viscosity essential for disk structure and evolution
- Molecular viscosity ν_m and mean free path λ_m:
- Example: r=10 AU, n=10^{12}cm^{-3}, c_s=0.5 km/s

$$\nu_m \sim \lambda_m c_s$$

$$\lambda_m = \frac{1}{n\sigma_m}$$

$$\sigma_{H_2} \simeq 2 \cdot 10^{-15} \mathrm{cm}^2$$

$$\nu_m = 2.5 \cdot 10^7 \mathrm{cm}^2 \mathrm{s}^{-1}$$

$$\tau_\nu \simeq \frac{r^2}{\nu} \quad \rightarrow \quad \tau_\nu \simeq 3 \cdot 10^{13} \mathrm{years}$$

- Reynolds Number Re: Disk are highly turbulent

$$\rightarrow \quad \mathrm{Re} = \frac{UL}{\nu_m} = 10^{10}$$

Shakura-Sunyaev Disks

- Define a *turbulent* or *effective viscosity*
- Simple description of such a viscosity, based on scaling arguements (Shakura & Sunyaev 1973)
- Largest scale of an eddy is scale heigth h
- Velocity of the order of sound velocity, otherwise shock and rapid dissipation
- Shakura-Sunyaev α-parameter, so-called α-disks, α<1

$$\nu = \alpha c_s h$$

- Different attempts for calculating this parameter, radial dependence of $\alpha=\alpha(r)$ is unknown
- Solutions of numerical computations are not satisfying observational constraints

Magnetorotational instability (I)

$$\frac{d}{dr}\left(r^2\Omega\right) < 0$$

$$\frac{d}{dr}\left(\Omega^2\right) < 0$$

- Instability of rotating disks with $M_{disk}/M_{*} < h/r$ from linear stability analysis due to Rayleigh
- Keplerian disks are Rayleigh-stable, i.e. angular momentum increases with radius
- Magnetic fields provide additional degrees of freedom, violently destabilize disks
- Weakly magnetized fluids are unstable if angular velocity decreases with radius, so-called Magnetorotational instability (=MRI)
- Keplerian disks with weak magnetic fields are MRI-unstable
- MRI requires lengthy algebra (e.g. Balbus & Hawley, 1998) due to MHD-equations

Magnetorotational instability (II)

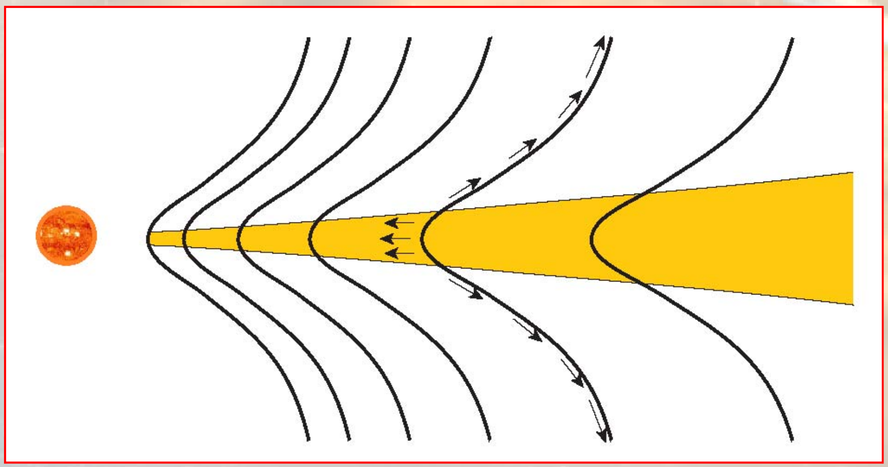
$$\beta = \frac{8\pi P}{B^2}$$

$$\beta > \frac{2\pi^2}{3}$$

- MRI requires a differentially rotating disk
- Small perturbations on magnetic field lines are sheared by differential rotation
- Magnetic tension separates the fluid elements by further angular momentum transport
- The MRI has large growth rates of the order of one orbital period
- Numerical simulations on the non-linear behavior of MRI leads to a state of sustained MHD turbulence
- Radial and azimuthal fluctuations in velocities can be averaged and interpreted as α–term, connect small-scale fluctuations to global angular momentum transfer

$$\alpha = \left\langle \frac{\delta u_r \delta u_\varphi}{c_s^2} - \frac{B_r B_\varphi}{4\pi\rho c_s^2} \right\rangle \sim 10^{-2}$$

Magnetic disk winds



$$T_{\text{mag}} = \frac{B^s_\varphi B^s_z}{2\pi}$$

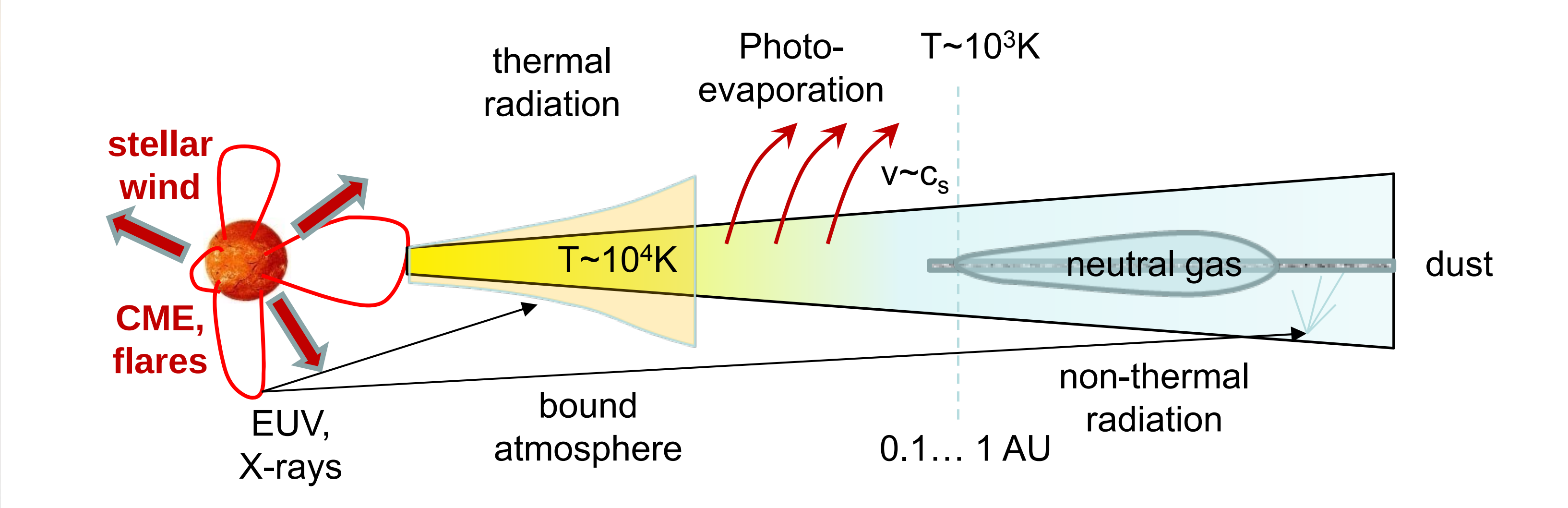
$$|u_{r,\text{mag}}| = \frac{B^s_\varphi B^s_z}{\pi \Sigma \Omega}$$

$$u_{r,\text{vis}} = -\frac{3}{2}\frac{\nu}{r}$$

$$\frac{|u_{r,\text{mag}}|}{|u_{r,\text{vis}}|} \sim \frac{B^s_\varphi B^s_z}{B_r B_\varphi} \left(\frac{h}{r}\right)^{-1}$$

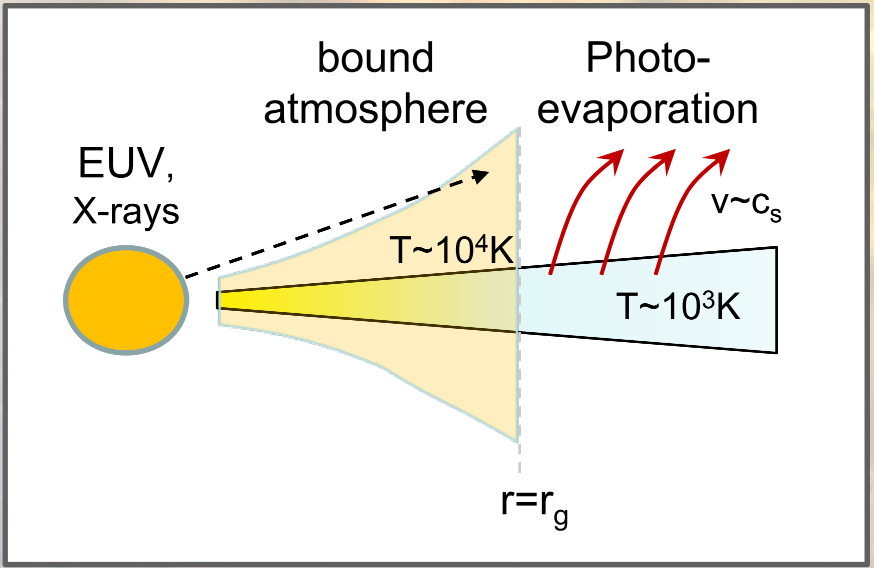
- Magnetic fields connect the disk with the external medium
- Torques T_{mag} on the disk surface (per unit area) lead to a radial inflow $u_{r,mag}$
- Compare this velocity with the α of MRI-generated radial inflow
- The global magnetic fields are much more efficient to reduce the angular momentum than internal redistribution by MRI
- BUT: Topology of magnetic field unclear, strong fields suppress MRI and many open questions on disk winds

Layered disks



- Magnetic fields will play important role for disk evolution
- Inner parts hot enough for thermal ionisation
- Cosmic rays, X-rays and EUV can penetrate deep to provide a level of ionization
- Possible *dead zone* of neutral gas without coupling to magnetic fields

Photoevaporation



- UV-radiation heats the disk surface, E = hν > 13.6eV
- Define a radius r_g where sound speed equals orbital velocity

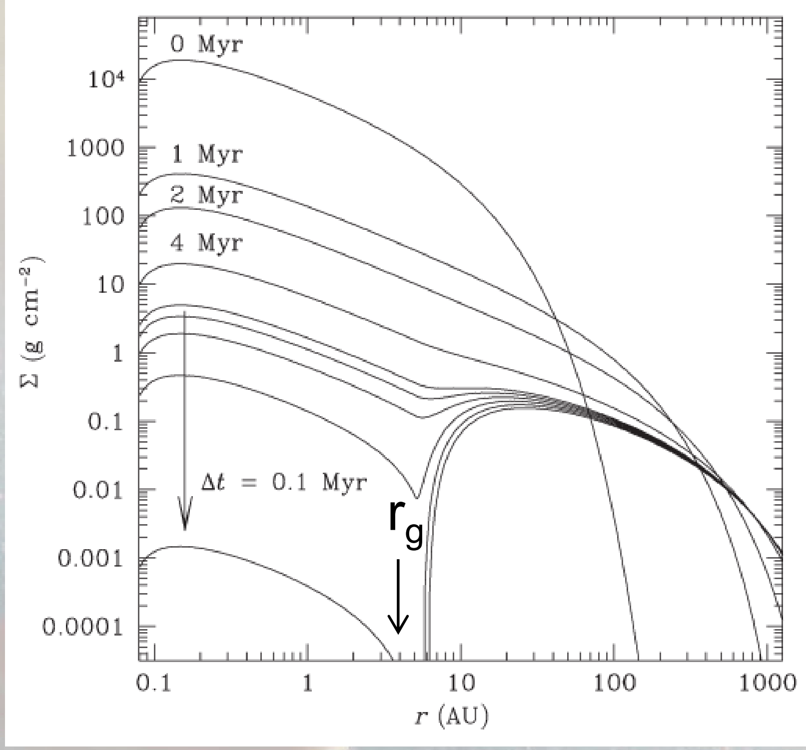
$$r_g = \frac{GM_*}{c_s^2} = 8.9 \left(\frac{M_*}{[M_\odot]}\right) \left(\frac{c_s}{[10\,\mathrm{km/s}]}\right)^{-2} \mathrm{AU}$$

$$\dot{\Sigma}_{\mathrm{wind}} \simeq 2\rho(r) c_s$$

- Loss of mass due to ionisation at $r>r_g$, material is unbound and escapes
- Numerical simulation lead to

$$\dot{M}_{\mathrm{wind}} \simeq 1.6 \cdot 10^{-10} \left(\frac{\Phi}{[10^{41}\mathrm{s}^{-1}]}\right)^{1/2} \left(\frac{M_*}{[M_\odot]}\right)^{1/2} M_\odot\,\mathrm{yr}^{-1}$$

Viscous evaporation



Clarke et al. (2001)

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial(\nu\Sigma r^{1/2})}{\partial r}\right] + \dot{\Sigma}_{\rm wind}(r,t)$$

- Simple model with mass loss, e.g. r_g=5 AU
- Disk evolves in three phases
  - Mass loos decouples inner and outer disk, gap around r_g
  - Inner disk evolution on viscous time scale ~10^5 years, accretion on central object
  - Inner disk becomes optically thin, UV-flux illuminates inner edge of outer disk, total dispersal within 10^5 years

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