On the dynamical stability of the solar system

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Introduction

- chaotic behavior of solar system (Laskar 1989)
- 4 terrestial planets: 5 mio. yrs Lyapunov-time (Laskar 1989)
- mass ratios of the planets to the sun are much larger than those required from KAM

◊ Is the solar system dynamically stable?

Introduction

- long-term numerical integration of the full solar system over 20 Gyrs
- dynamically allowed evolutions in which the planetary orbits become unstable
- effects of general relativity on the dynamical stability
- dynamical lifetime of Uranus

chaotic motion - Lyapunov time

$$\gamma = \lim_{N \to \infty} \sum_{k=1}^{N} \ln (s_k / s_0)/(N\Delta t)$$

s_0 ... separation vector (150 m, radially outward)

Δt... 10.000 yr

N... 100

$1/\gamma$... Lyapunov time

$s/s_0 = e^{(\gamma \Delta t)}$

Table 1: Lyapunov exponents and times for the Solar System

Planet	Lyapunov exp. (yrs 10^{-1})	Lyapunov time (yrs)
Mercury	7.32029 10^{-7}	1.36607 10^{6}
Venus	1.38561 10^{-7}	7.21703 10^{6}
Earth	2.07484 10^{-7}	4.81964 10^{6}
Mars	2.22353 10^{-7}	4.49736 10^{6}
Jupiter	1.19528 10^{-7}	8.36623 10^{6}
Saturn	1.56875 10^{-7}	6.37452 10^{6}
Uranus	1.33793 10^{-7}	7.47423 10^{6}
Neptune	1.49602 10^{-7}	6.68440 10^{6}

chaotic motion - Lyapunov time

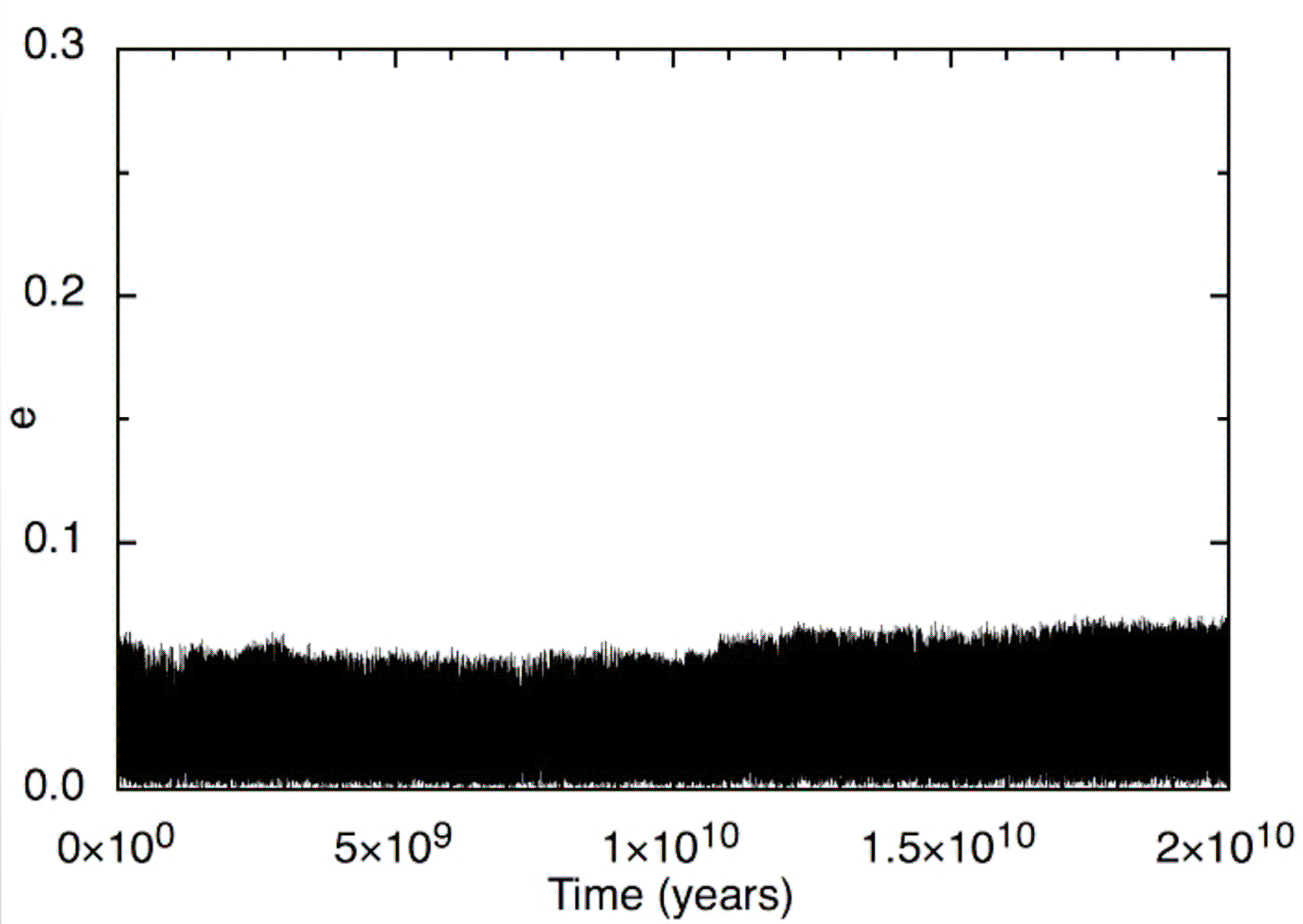
- Lyapunov time - some 10^6 yrs

◊ Why long-term integrations?

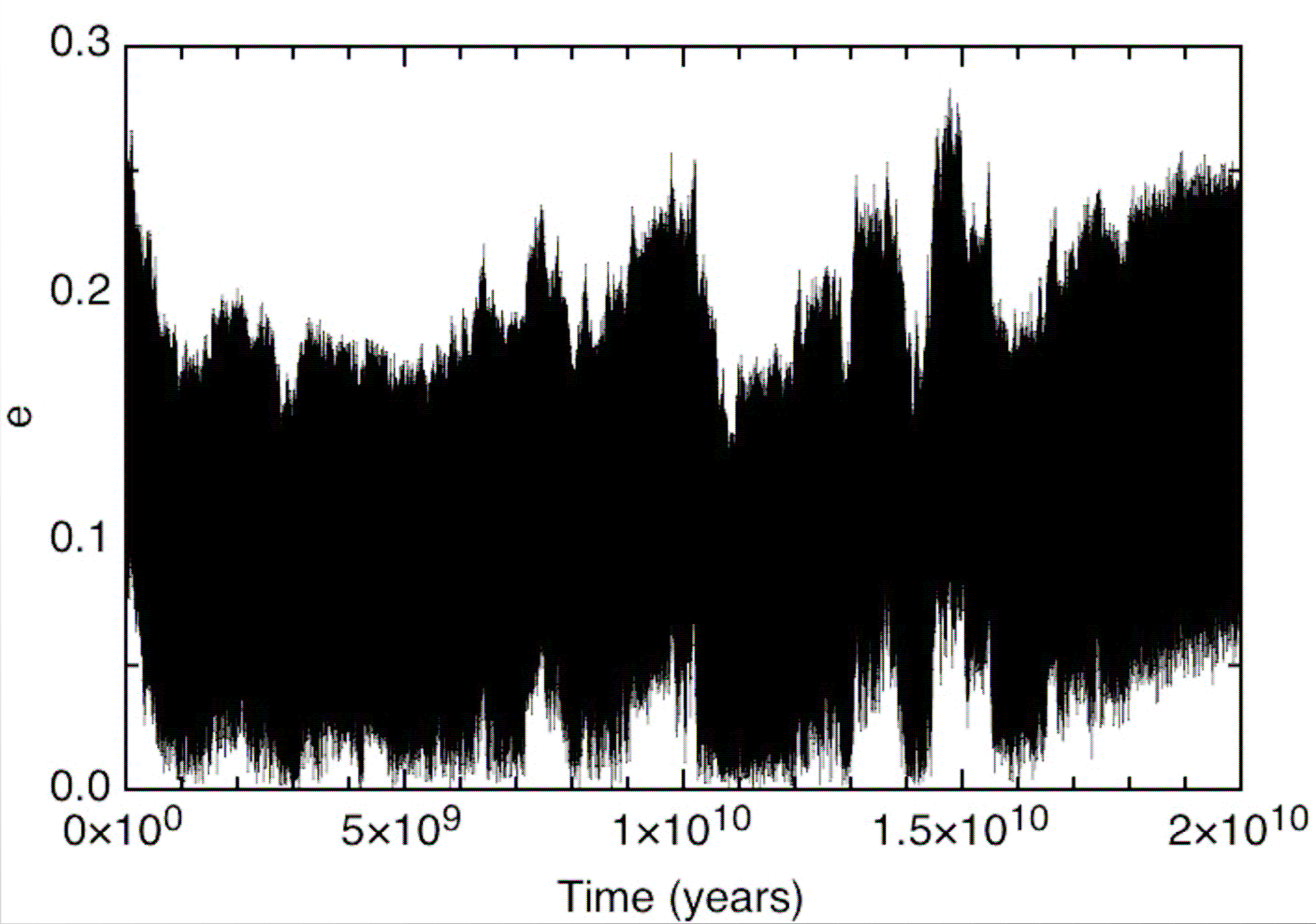
They give probalistic evaluations of the solar system's future behavior.

direct long-term integration

- start: current configuration
- timespan: over 20 Gyrs
- timestep: 8 days
- conservation of total energy:
  $\Delta E/E < 10^{-7}$
- conservation of total angular momentum:
  $\Delta L/L < 10^{-9}$



Eccentricity of earth as function of time



Eccentricity of mercury as function of time

direct long-term integration

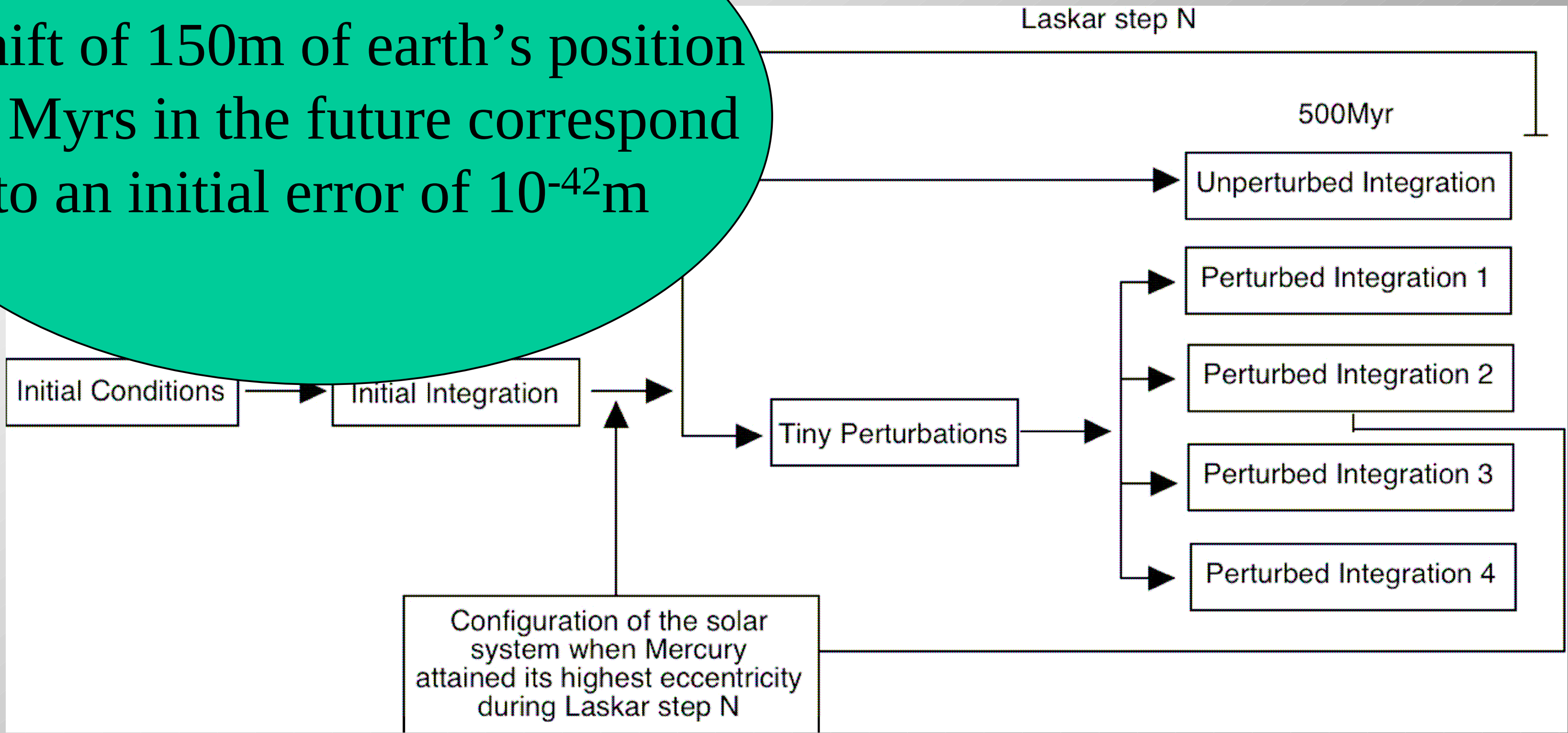
◊ solar system seems to be stable over its lifetime?

but: its only <u>one</u> possible trajectory! (Laskar 1994)

◊ better: any timescale for occurring instabilities might be long

The Laskar - method

A shift of 150m of earth's position 500 Myrs in the future correspond to an initial error of 10^{-42}m



The Laskar - method

numerical induced chaos - limiting factor:

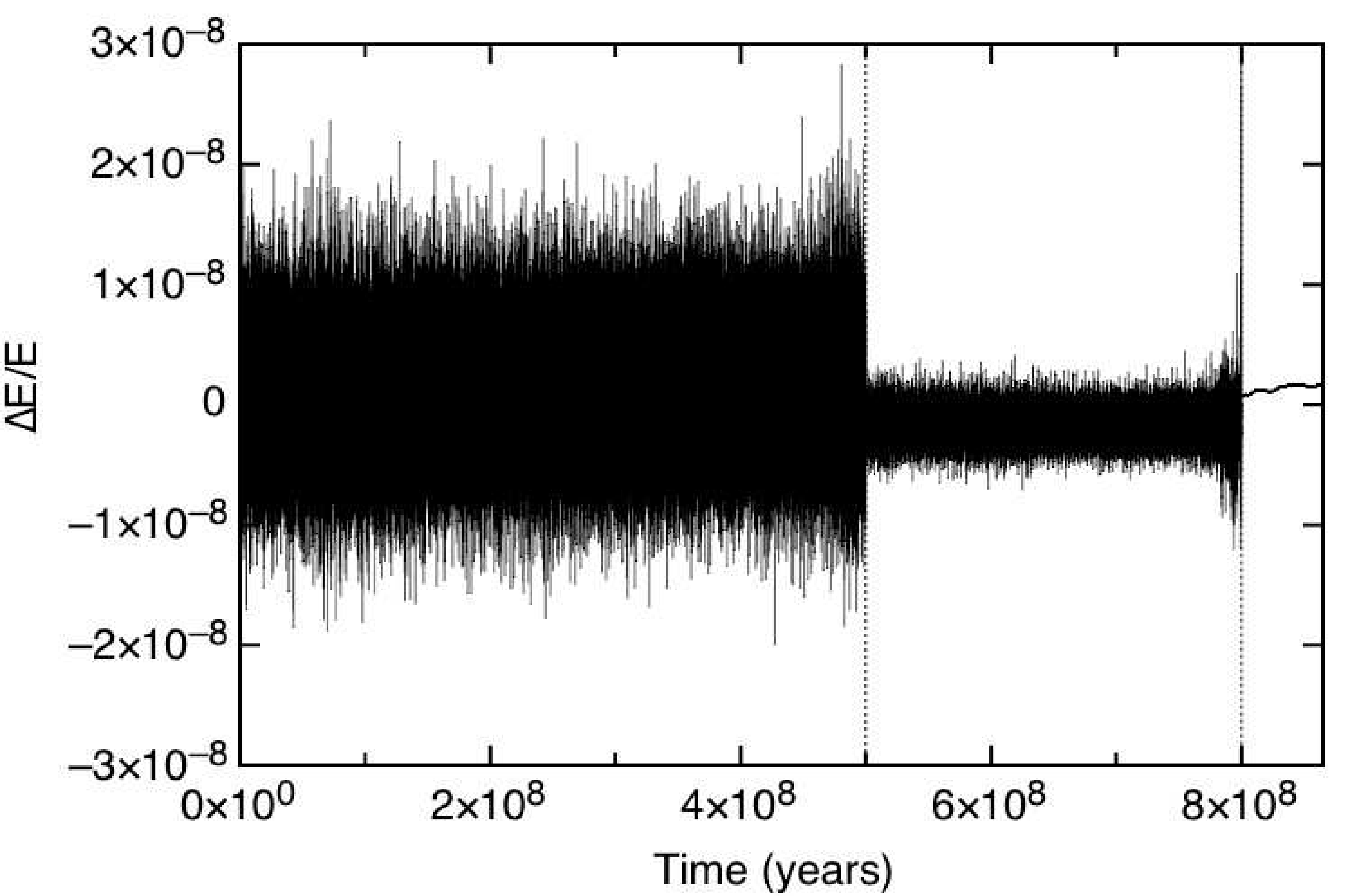
accurately resolving Mercury’s orbit(Yoshida,1993)

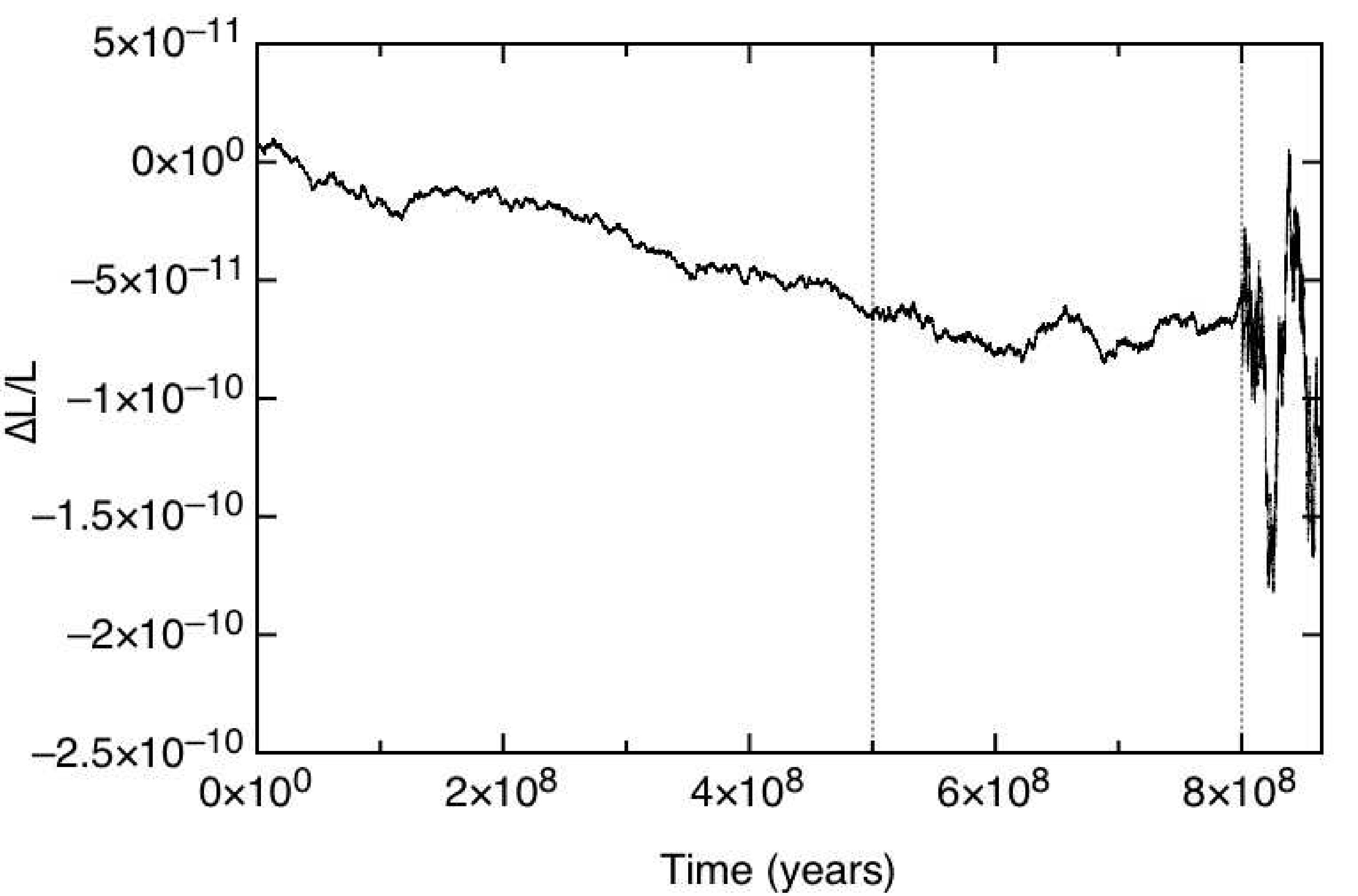
◊ conservation of energy and ang.momentum

◊ criteria for this work:

$\Delta E/E \sim 10^{-8}$ and $\Delta L/L \sim 10^{-10}$

◊ variation of timestep form 3 to 1.2 days to correct the violation of these criterias


3×10⁻⁸
2×10⁻⁸
1×10⁻⁸
0
−1×10⁻⁸
−2×10⁻⁸
−3×10⁻⁸
ΔE/E
0×10⁰
2×10⁸
4×10⁸
6×10⁸
8×10⁸
Time (years)


5×10^−11
0×10^0
−5×10^−11
−1×10^−10
−1.5×10^−10
−2×10^−10
−2.5×10^−10
ΔL/L
0×10^0
2×10^8
4×10^8
6×10^8
8×10^8
Time (years)

The Laskar - experiment

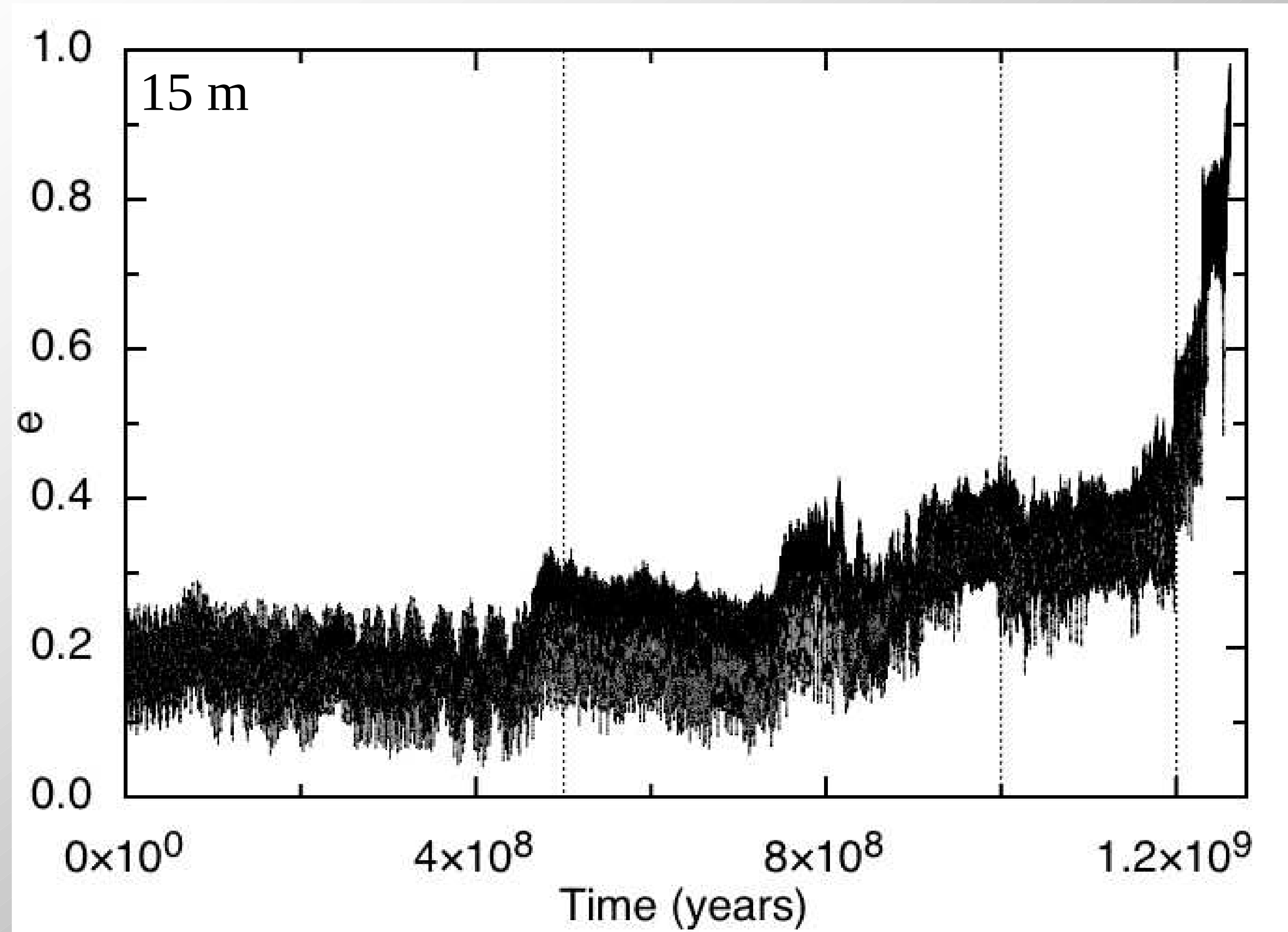
- 2 experiments: 150m and 15 m

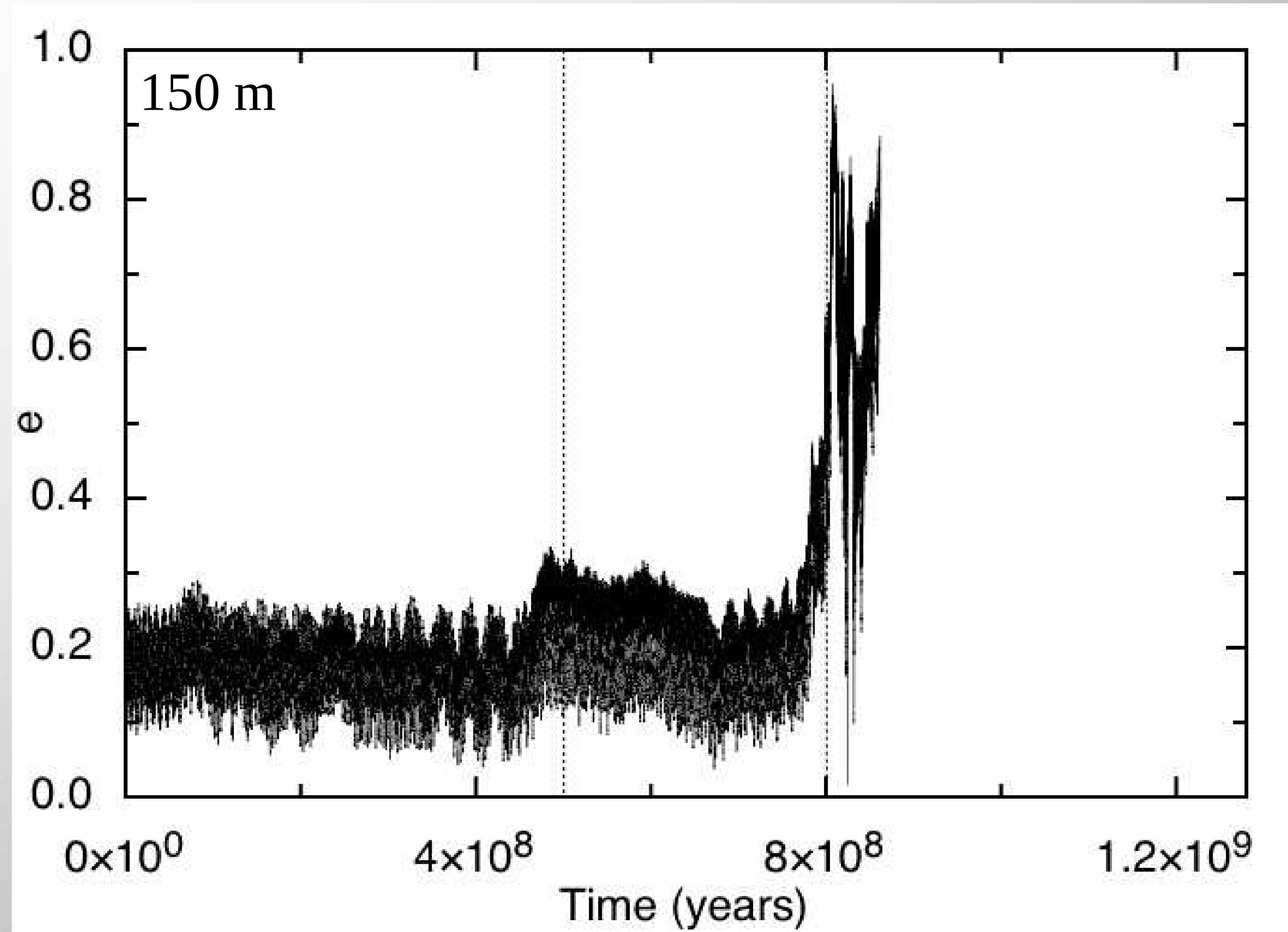
Step	Time interval	Endpoint
1	0-500	0.2907
2	500-797	0.4391
3	797-862	0.8257

◊ Collision with Venus

Step	Time interval	Endpoint
1	0-500	0.2907
2	500-994	0.4139
3	994-1207	0.4874
4	1207-1261	0.9751

◊ Collision with Sun


15 m
1.0
0.8
0.6
0.4
0.2
0.0
e
0x10^0
4x10^8
8x10^8
1.2x10^9
Time (years)


150 m
1.0
0.8
0.6
0.4
0.2
0.0
e
0x10^0
4x10^8
8x10^8
1.2x10^9
Time (years)

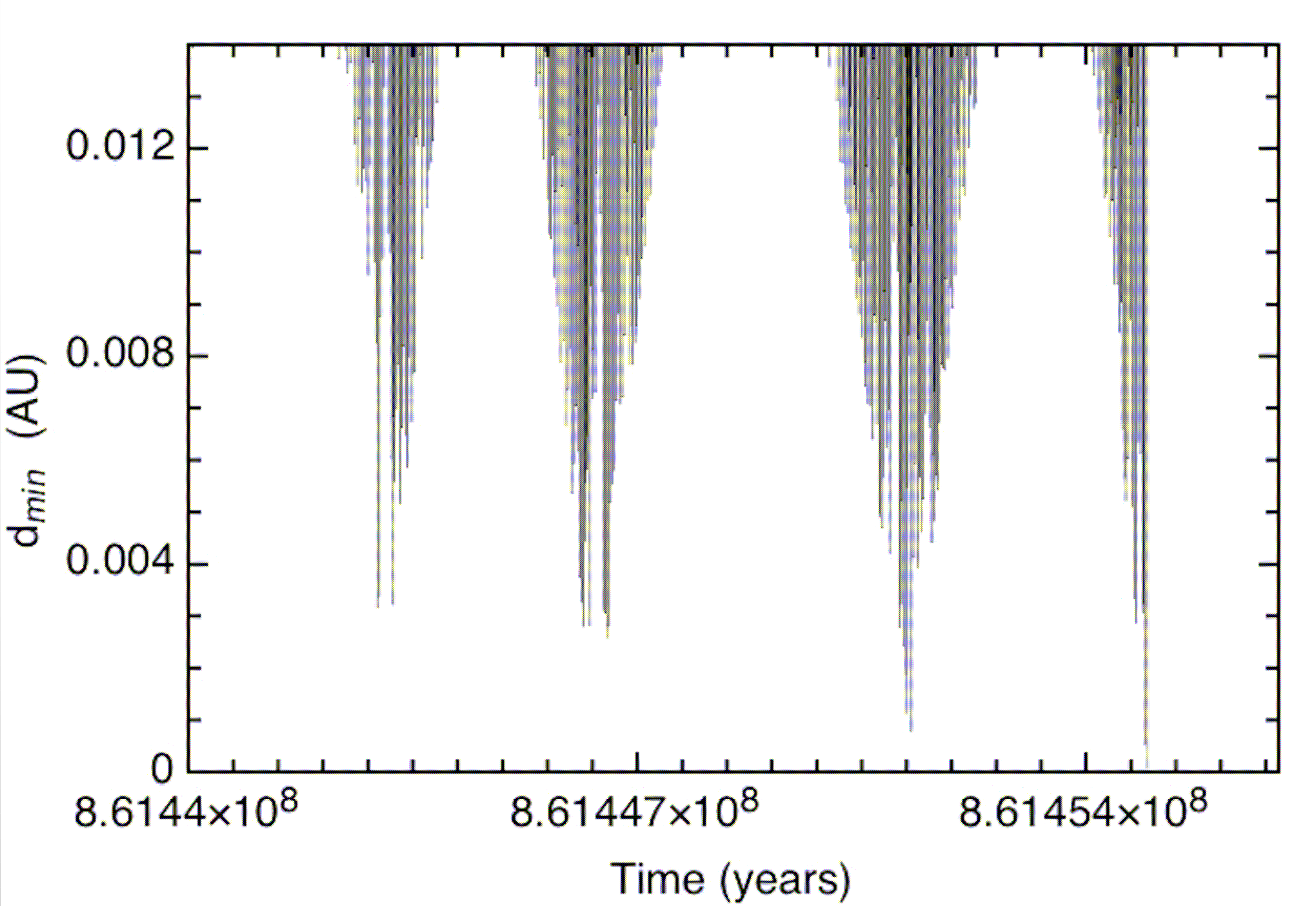
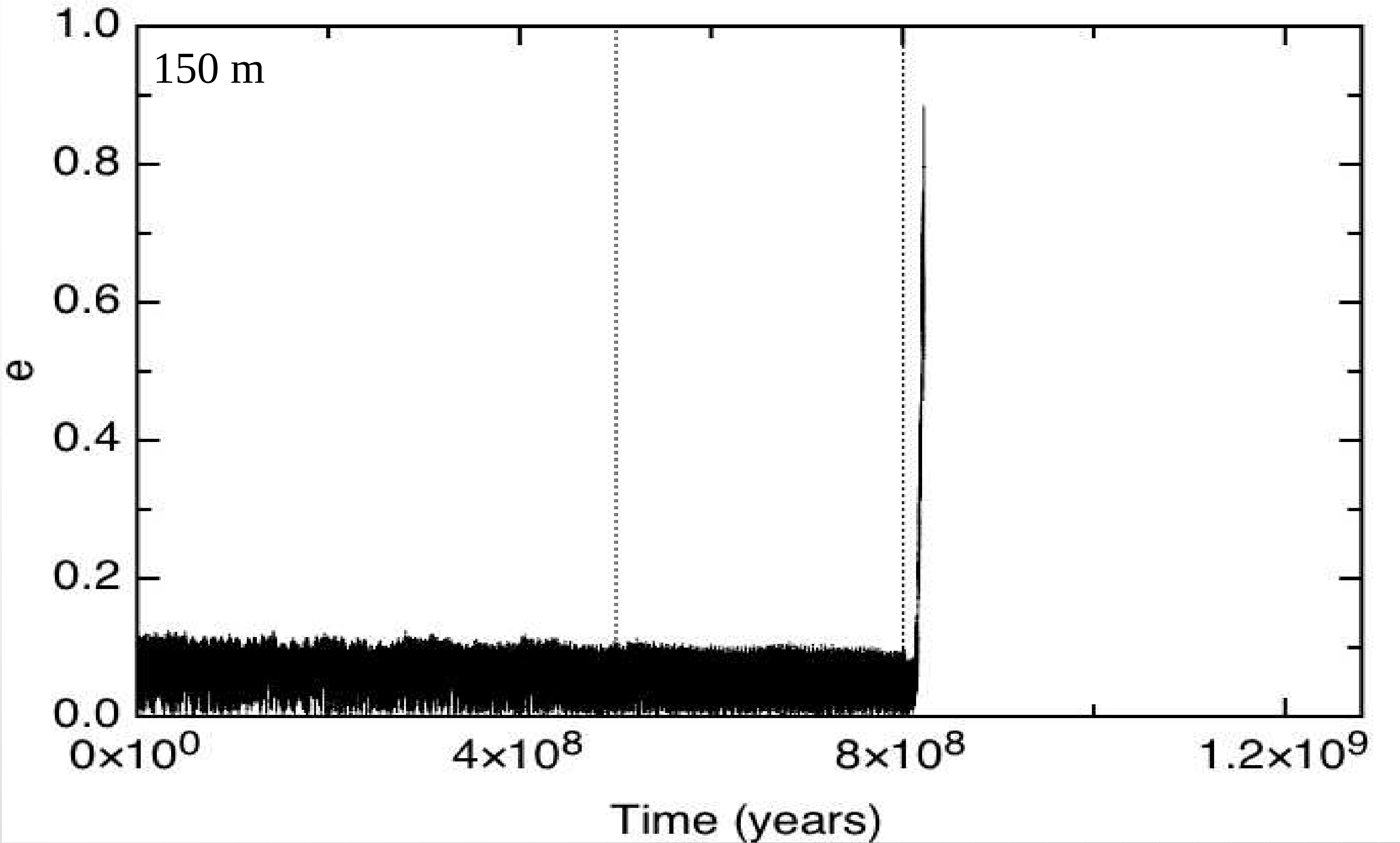


Fig. 10. The minimal distance of approach during a series of close encounters between Mercury and Venus as a function of time. The collision takes place at t ~ 861.455Myr, when dmin = 5.5561 x 10^{-5} AU < r_venus + r_mercury = 5.6762 x 10^{-5} AU.



Evolution of Mars' eccentricity in the 150 m experiment at t = 822 Myrs - also Mars' semimajor axis increased and after it reached a distance > 100 AU, Mars was assumed to be ejected fromsolar system

The Laskar - experiment

- Mercury shows different ways of evolution
- Mercury tends to switch from regular to irregular motion

◊ Exists any association with timesteps or integral algorithm?

◊ Reintegration of a 22 Myr time interval where Mercury became unstabel

The Laskar - experiment

150 m

symplectic algorithm

0.5 days time step

- 22 Myrs

Bulirsch-Stoer algorithm

0.5 days time step

15 m

symplectic algorithm

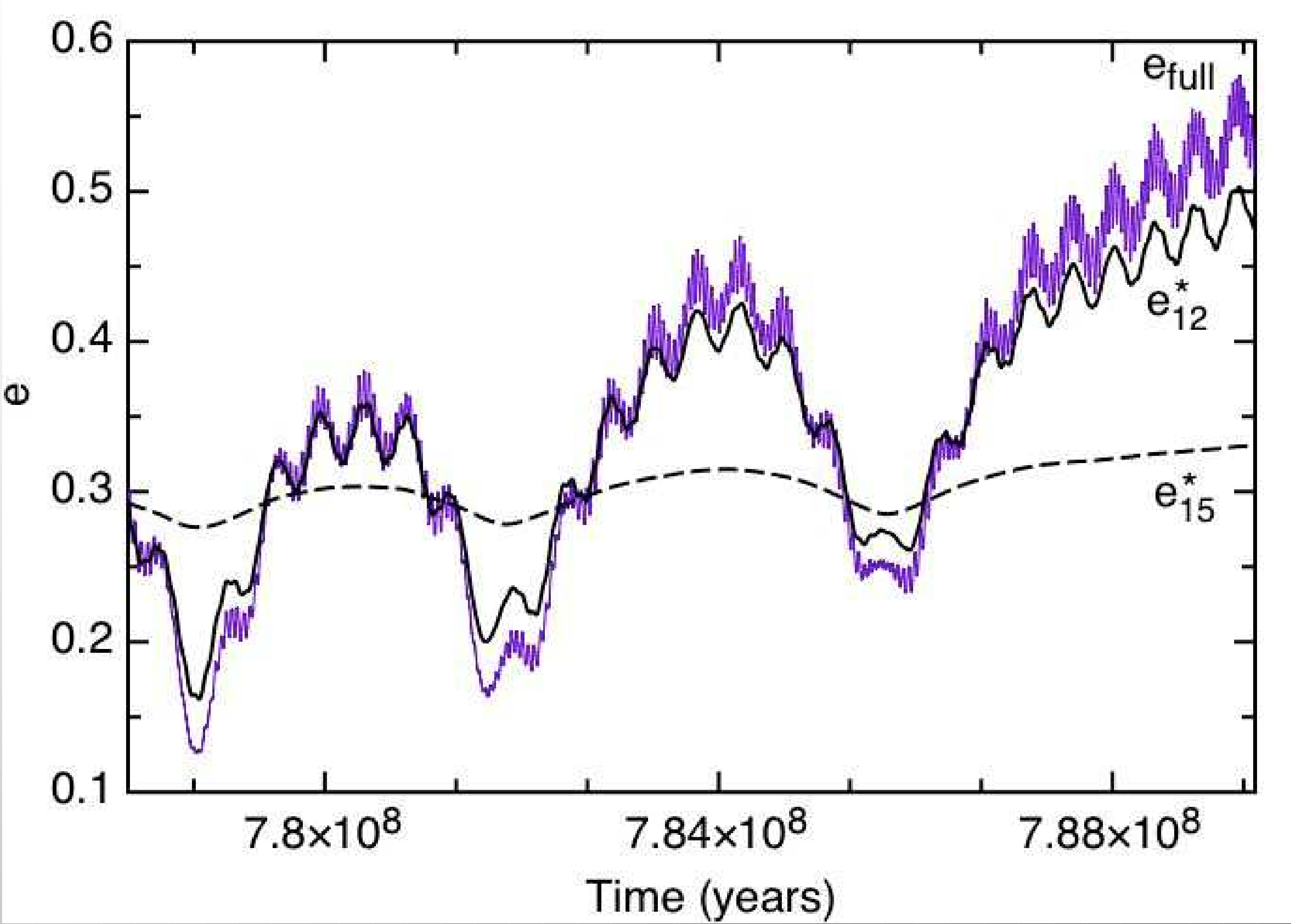
0.5 days time step

- 20 Myrs

Bulirsch-Stoer algorithm

0.5 days time step

almost identical increase of eccentricity as in the primary solution


0.6
0.5
0.4
0.3
0.2
0.1
e
e_{full}
e^*_{12}
e^*_{15}
7.8×10^8
7.84×10^8
7.88×10^8
Time (years)

The Laskar - experiment

Starting at 778 Myrs for the 150 m:

- adding perturbations of 15 m and 150 m in four directions and integrating over 22 Myrs

◊ no changes

Starting at 1190 Myrs for the 15 m:

- adding perturbations of 15 m and 150 m in four directions and integrating over 20 Myrs

◊ no changes

The Laskar - experiment

- until 780 Myrs Mercury’s eccentricity varies within a narrow and well-defined range
- shortly thereafter the eccentricity increases and leads to collisions,.....

◊ secular resonances

secular resonances

stable motion versus unstable motion

$\langle R_i^{sec} \rangle = n_i a_i^2 \{0.5 A_{ii} e_i^2 + 0.5 B_{ii} I_i^2 + \Sigma [A_{ij} e_i e_j \cos(\varpi_i - \varpi_j) + B_{ij} I_i I_j \cos(\Omega_i - \Omega_j)] \}$

$R_i = \Sigma [(G m_j)/(|r_j - r_i|) - G m_j(r_j \times r_i)/(r_j^3)]$

secular resonances

The dominant frequencies of the secular disturbing function can be identified by Fourier-analyzing the numerically computed time-series for Mercury's full disturbing function.

$$g_i = \langle \dot{\varpi}_i \rangle$$

stable motion

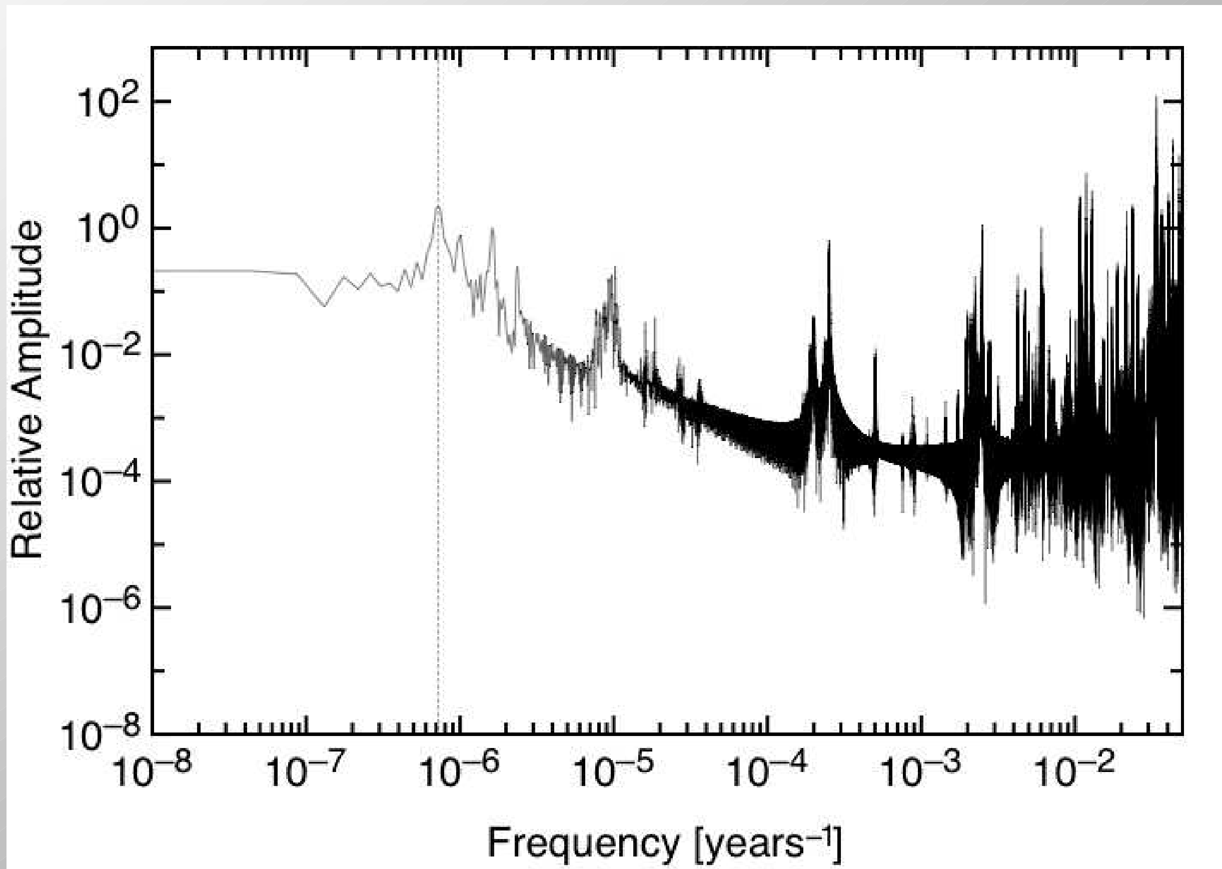
$(g_1 - g_5) \sim 0.9389'' \, yr^{-1}$

unstable motion

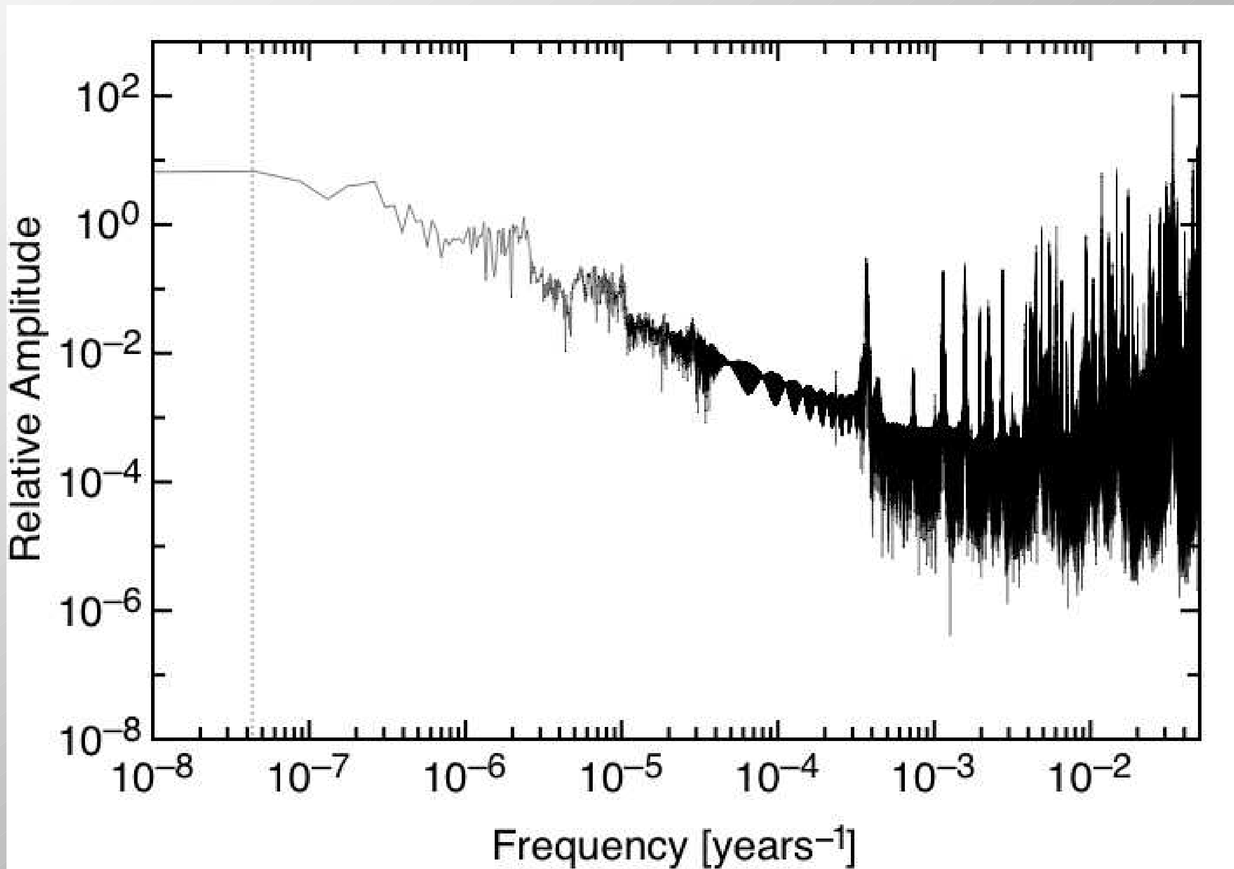
$(g_1 - g_5) \sim 0.0538'' \, yr^{-1}$

(Laskar 1990)

secular resonances - stable motion



secular resonances - unstable motion



secular resonances

Classical Laplace-Lagrange secular solution for Mercury’s ecc. vector:

$h_1 = e_1 \sin \varpi_1$

$k_1 = e_1 \cos \varpi_1$

secular resonances

$h_1 = e_{1f} \sin(g_1 t + \beta_1) - \Sigma[(\nu_j /(g_1 - g_j)) \sin(g_j t + \beta_j)]$

$k_1 = e_{1f} \cos(g_1 t + \beta_1) - \Sigma[(\nu_j /(g_1 - g_j)) \cos(g_j t + \beta_j)]$

small divisors indicate large influence !!

secular resonances

classical value of Mercury's proper frequ.:

g_1 = 5.4058"

during Mercury's evolution of ecc.:

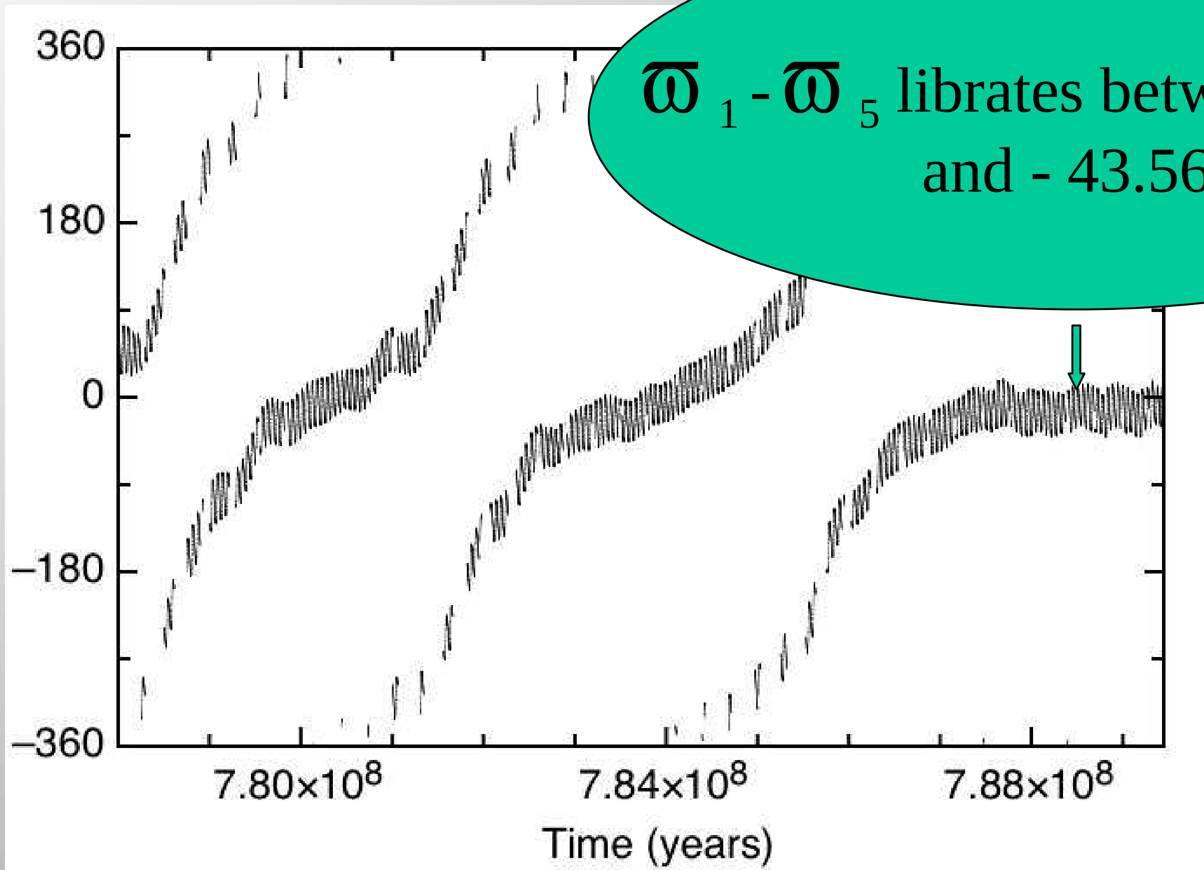
g_1 = 4.9273"

g_j = 4.24354"

◊ linear secular resonances

◊ large variations in the ecc. vector

secular resonances



$\varpi_1 - \varpi_5$ librates between +19,8 º and - 43.56º

secular resonances

$$de_{15}/dt = A_{15}\, e_5\, (1-e^2)^{0.5} \sin(\varpi_1 - \varpi_5)$$

$$g_1 \Rightarrow \langle \dot{\varpi}_1 \rangle \quad \text{and} \quad g_5 \Rightarrow \langle \dot{\varpi}_5 \rangle$$

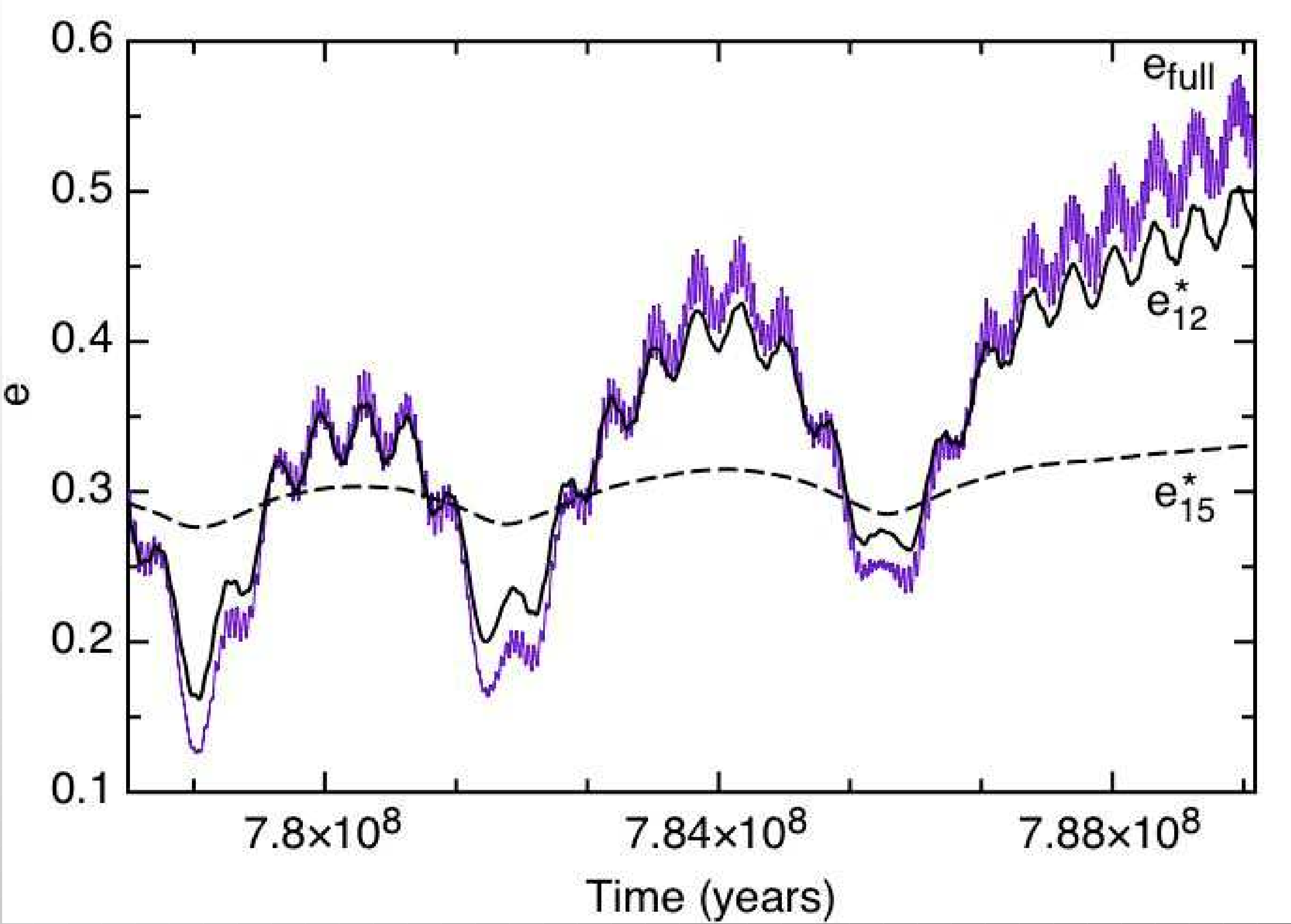
◊ $(g_1 - g_5)$ secular resonance responsible for the climb in Mercury's eccentricity.

secular resonances

Influence of Venus:

$$g_2 \not\Rightarrow \langle \dot{\varpi}_2 \rangle \text{ (Laskar,1990)}$$

◊ e_{12} constructed as before

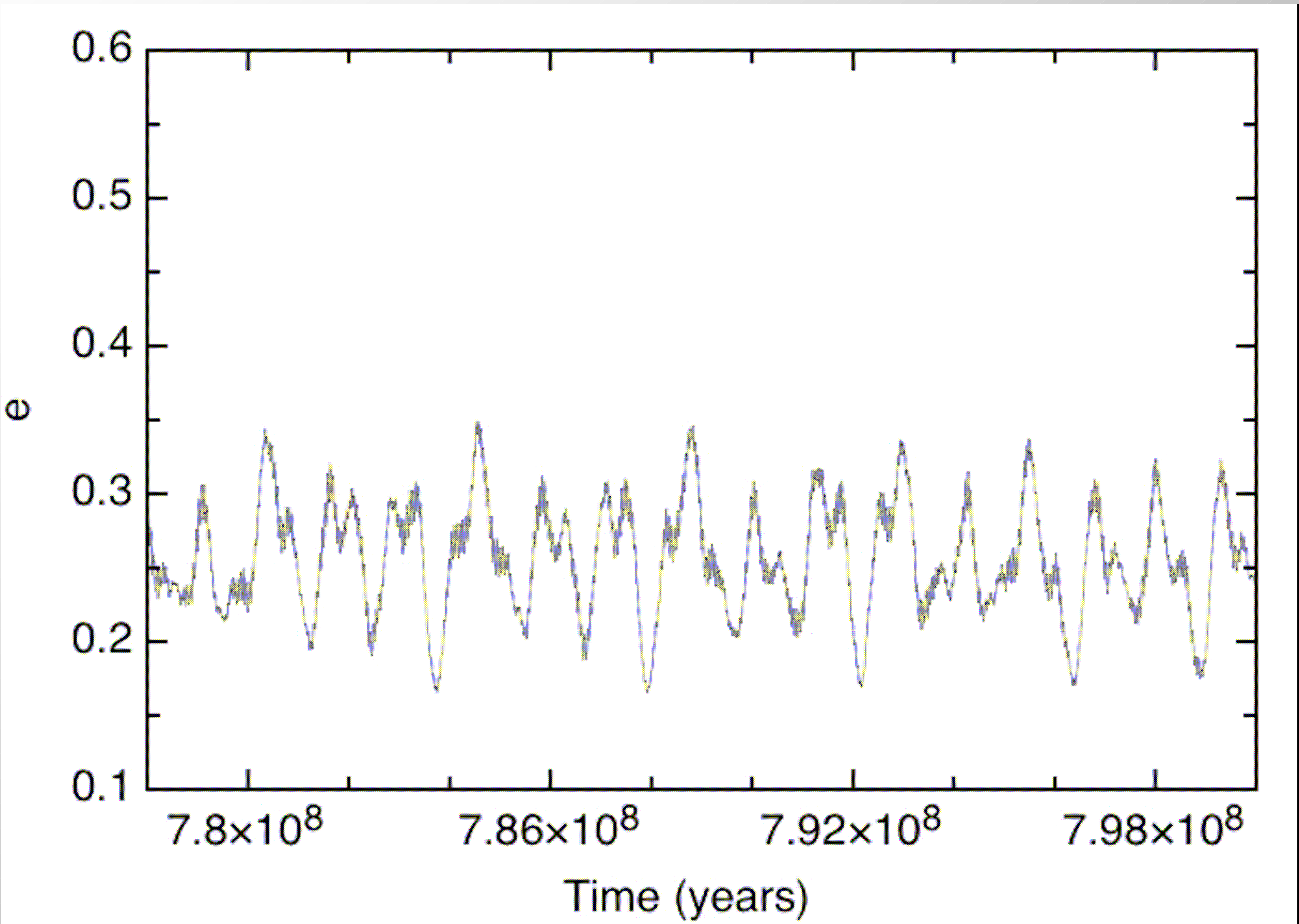

0.6
0.5
0.4
0.3
0.2
0.1
e
e_{full}
e^*_{12}
e^*_{15}
7.8×10^8
7.84×10^8
7.88×10^8
Time (years)

relativistic influence

- adding 0.43“ yr^{-1}
- $g_1 - g_5$ gets about 40 % higher

◊ Mercury enters later in the linear secular resonance with Jupiter

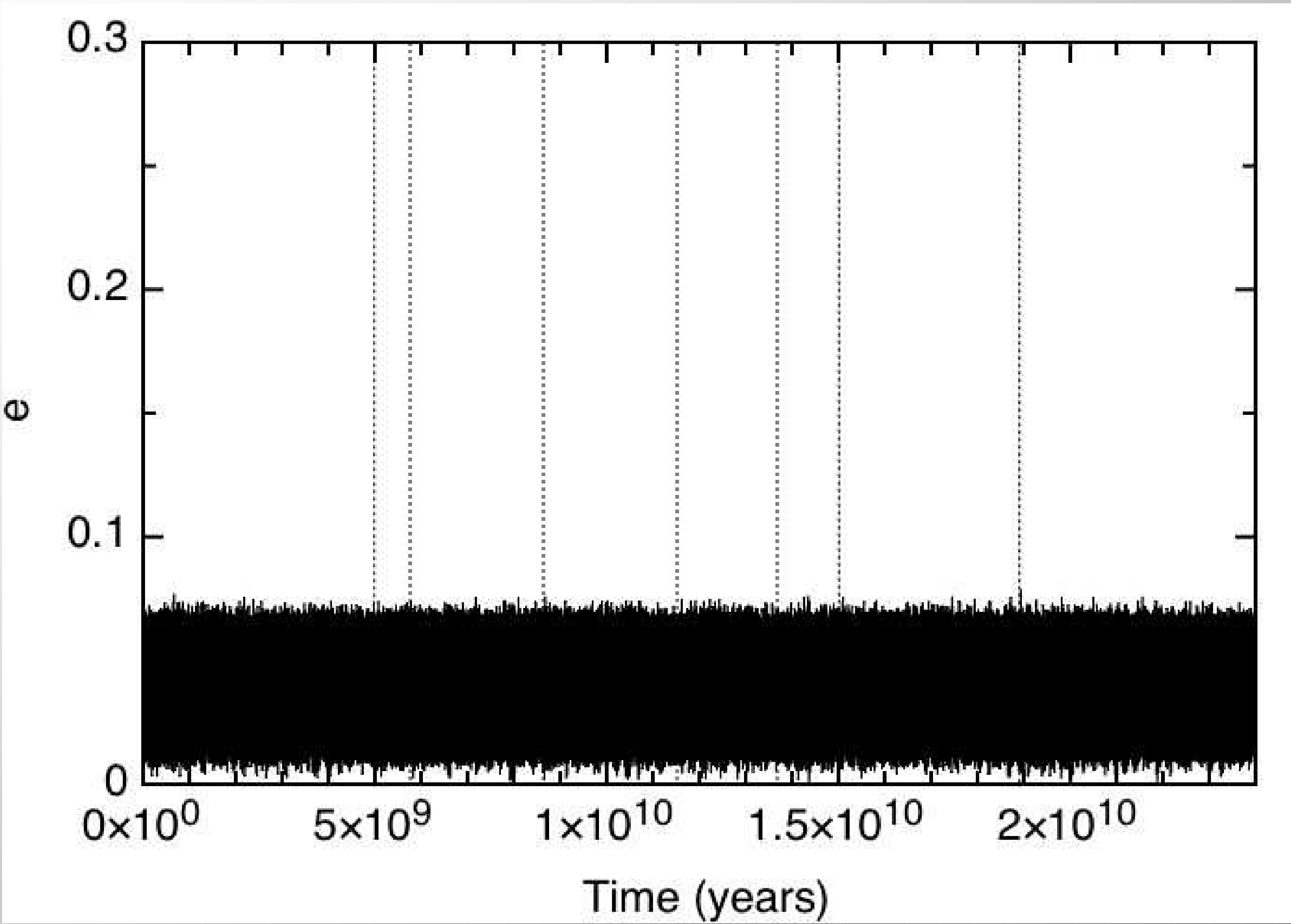
◊ relativistic influence stabilizes Mercury ’s orbit


0.6
0.5
0.4
0.3
0.2
0.1
e
7.8×10^8
7.86×10^8
7.92×10^8
7.98×10^8
Time (years)

Uranus dynamical lifetime

- Laskar experiment for Uranus
- sun (+4 terrestrial planets) and the 4 gasgiants
- Jupiter was repositioned 1500m
- Integration intervals: 5 Gyrs

◊ Uranus ecc. never exceeded 0,078 !!


0.3
0.2
0.1
0
e
0×10^0
5×10^9
1×10^10
1.5×10^10
2×10^10
Time (years)


Merry Christmas
one and all
x x x
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