STABILITY OF HYPOTHETICAL TROJAN PLANETS IN EXOPLANETARY SYSTEMS

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Abstract The stability of hypothetical Trojan planets in exoplanetary systems is investigated. In the model of the planar three-body problem, corresponding to a gravitational system of a star, a giant planet and a Trojan planet, the stability regions for the Trojan planet around the Lagrangian point L_4 are determined depending on the mass of the two planets and the initial eccentricity of the orbit of the giant planet. The results indicate that in exoplanetary systems with one giant planet of several Jupiter-masses, a Trojan planet up to one Jupiter-mass can exist in stable motion around L_4.

Keywords: Trojan exoplanets – Stability

1. Introduction

The possible existence and stability of Trojan planets in exoplanetary systems have been the subject of several recent discussions. It is well known that Trojan asteroids exist in the Solar System in great number. It can be expected that Trojan-type objects exist also in exoplanetary systems. Laughlin and Chambers [3] outlined a possible formation mechanism of Trojan planets in protoplanetary accretion discs. They also discussed the question of detectability of extrasolar Trojan planets. According to their results two planets with masses comparable to the mass of Jupiter or Saturn around a solar-mass star can perform stable tadpole-type librations about the Lagrangian points L_4 or L_5 of the system. Pairs of Saturn-mass planets can also execute horseshoe orbits around a solar-mass star, but this is not possible for Jupiter-mass pairs. A pair of planets both in tadpole and horseshoe-type orbits induce a characteristic pattern in the radial velocity component of the central star that could be detected. Nauenberg [5] determined numerically the nonlinear stability domain of the triangular Lagrangian solutions in the general three-body problem as a function of the eccentricity of the orbits and the Routh's mass parameter. This study indicates that there is a wide range of Jupiter-size planetary masses (including brown dwarfs) and eccentricities for which such solutions could exist in exoplanetary systems.

Most of the known exoplanets are gaseos giant planets having large masses of the order of or several Jupiter-masses. The search for small terrestrial-like planets with solid surface is an outstanding aim of several ongoing and future research projects. It is an important question, whether Earth-like planets can exist in the habitable zone (HZ) of exoplanetary systems. If there is a giant planet in the HZ of a system, the existence of another planet there is unlikely. However, as Menou and Tabachnik [6] noted, terrestrial planets could exist at the stable Lagrangian points L_4 or L_5 of the giant planet moving in the HZ.

Érdi and Sándor [2] studied this possibility in detail, investigating five exoplanetary systems (HD 17051, HD 28185, HD 108874, HD 27442, and HD 114783) in which the only known giant planet moves in the HZ. By using the model of the elliptic restricted three-body problem they determined numerically the region around L_4 of each system where stable tadpole-type motion is possible. In [2] four other systems (HD 150706, HD 177830, HD 20367, and HD 23079) were also studied in which the orbit of the giant planet is partly outside the HZ due to its large eccentricity. It has been shown that in all studied systems there is an extended stability region around L_4, whose extent depend on the mass and the orbital eccentricity of the giant planet. It is possible that Trojan exoplanets of negligible mass exist in these systems.

Dvorak et al. [1] also studied three exoplanetary systems in which a giant planet moves close to the HZ in low eccentricity orbit. They determined the size and the structure of the stability region around L_4 and L_5 and pointed out that the stability region shrinks significantly with the increase of the orbital eccentricity of the giant planet. It is possible that in all three systems a small Trojan planet could exist in stable orbits with moderate eccentricities.

In our previous study [2] we assumed that the fictitious Trojan exoplanet had negligible mass. In this paper we study the problem more generally, giving mass to the Trojan planet up to 1 Jupiter-mass and determine the regions of stability around L_4 in the model of the planar three-body problem.

2. Dynamical model and method of investigation

For the investigation of the nonlinear stability of orbits around L_4 we used the model of the planar three-body problem, corresponding to a gravitational system of a star, a giant planet and a Trojan planet, by assuming, as a first step of a more general stability study, that the orbits of the two planets are in the same plane.

To determine the dynamical character of the orbits we used the method of the relative Lyapunov indicators (RLI) developed by Sándor et al. [7], [8]. The RLI measures the difference in the convergence of the finite-time Lyapunov indicators to the maximal Lyapunov characteristic exponents of two initially very close orbits. The values of the RLI are characteristically several orders of magnitude larger for orbits in a chaotic region than in a regular domain. The method is extremely fast in establishing the ordered or chaotic nature of individual orbits, and therefore is very well applicable to explore the dynamical structure of the phase space. According to our experiments, gained in different dynamical problems, it is enough to integrate the two very close orbits for a few hundred times of the longest orbital period of the studied system. In the present investigation we integrated the orbits for 10^3 periods of the giant planet.

In our computations we used the following parameters and initial orbital elements.

- Mass of the central star: $m_0 = 1\, m_\odot$ (solar mass)
- Mass of the giant planet: $m' = 1$, 2, 3, 4, 5, 6, 7 m_J (Jupiter-mass)
- Initial orbital elements of the giant planet:
  - semi-major axis: $a' = 1$ AU
  - eccentricity: $e' = 0 - 0.30$, stepsize: $\Delta e' = 0.05$
  - argument of the pericentre: $\omega = 0$
  - mean anomaly: $M' = 0$
- Mass of the Trojan planet: $m = 0$, 1, 2, 3, 10, 100 m_E (Earth-mass) and 1 m_J
- Initial orbital elements of the Trojan planet:
  - semi-major axis: $a = 0.8 - 1.2$ AU, stepsize: $\Delta a = 0.001$ AU
  - eccentricity: $e = 0$
  - synodic longitude: $\lambda - \lambda' = 20^\circ - 180^\circ$, stepsize: $\Delta\lambda = 2^\circ$,

where λ and λ' are the mean orbital longitudes of the Trojan and the giant planet, respectively. (Initially $\lambda' = 0$, since $\lambda' = M' + \omega'$.)

We computed maps of dynamical stability around L_4 in the following way. Selecting a value of m', e' and m from the given sets, we changed the semi-major axis and the synodic longitude of the Trojan planet in the given intervals with the given stepsize and computed the values of the RLI for all resulting orbits. Then we represented the logarithm of the values of the RLI corresponding to each initial point on the (a, $\lambda - \lambda'$) plane on a black and white scale. In what follows we discuss the main characteristics of these maps. Some representatives of them are shown in Figures 1-8. Low RLI values (light regions) correspond to stable orbits, high RLI values (dark shades) indicate chaotic behaviour. The black background corresponds to escape or collision orbits with the giant planet. Considering that 7 values for the mass of both planets, and also 7 values for the initial orbital eccentricity of the giant planet were taken, altogether 343 maps were computed. These dynamical stability maps can be used to establish the stability region around L_4 in known exoplanetary systems with one giant planet.

3. Maps of dynamical stability

Fig. 1 shows the stability region around L_4 for $m = 0$, $m' = 1m_J$, $e' = 0$ (circular restricted three-body problem, with mass parameter $\mu = m'/(m_0 + m') \approx 0.001$). It can be seen that there is a central more stable region and going outwards a ring structure appears corresponding to higher order resonances between the short and long period components of the librational motion around L_4.

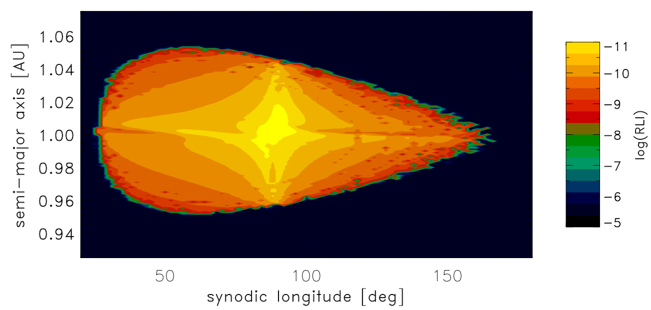

Figure 1. Structure of the stability region around L_4 in the circular restricted three-body problem ($m = 0$, $e' = 0$) for the mass parameter $\mu \approx 0.001$ ($m' = 1m_J$).

The computations show that increasing the mass of the giant planet, the stability region becomes shorter in the synodic longitude and wider in the semi-major axis. Near its edge the ring structure disrupts into a chain of islands. In Fig. 2, obtained for $m' = 2m_J$, both a ring and a chain of small islands can be seen. These islands are remnants of a former ring. The shrinking of the stability region with the increase of the mass of the giant planet is not monotonic, it reaches a minimum extension at $m' = 6m_J$ (Fig. 3), then it is larger again for $m' = 7m_J$ (not shown in the figures).

In the elliptic restricted three-body problem, when $m = 0$ and $e' \neq 0$, the structure of the stability regions is similar to that of the circular problem. Figs. 4 and 5 show two examples which are somewhat different from the general

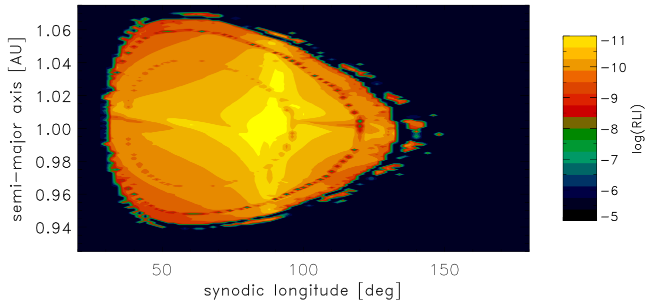

Figure 2. Structure of the stability region around L_4 in the circular restricted three-body problem ($m = 0$, $e' = 0$) for $\mu \approx 0.002$ ($m' = 2m_J$).

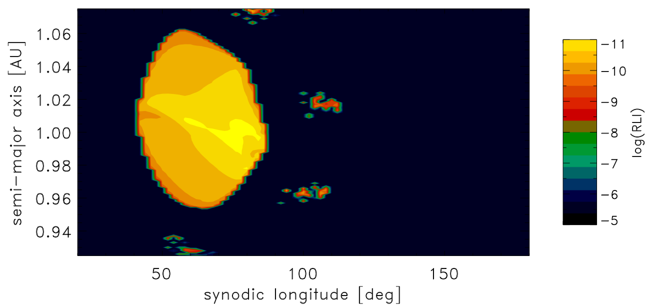

Figure 3. Structure of the stability region around L_4 in the circular restricted three-body problem ($m = 0$, $e' = 0$) for $\mu \approx 0.006$ ($m' = 6m_J$).

picture. Fig. 4, obtained for $e' = 0.1$, $m' = 4m_J$, exhibits a well structured stability region around L_4. In Fig. 5, obtained for $e' = 0.2$, $m' = 3m_J$, a compact stability region is present.

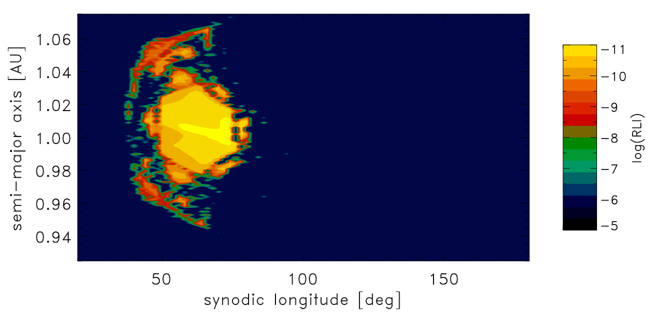

Figure 4. Structure of the stability region around L_4 in the elliptic restricted three-body problem for $e' = 0.1$, $\mu \approx 0.004$ ($m = 0$, $m' = 4m_J$).

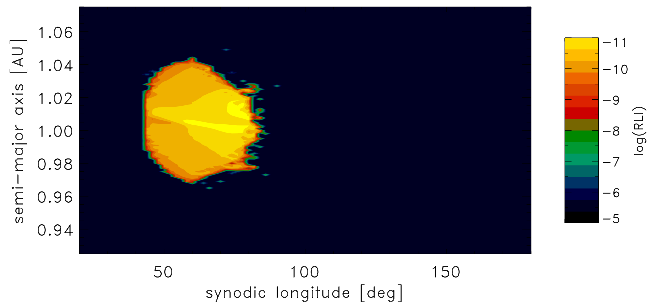

Figure 5. Structure of the stability region around L_4 in the elliptic restricted three-body problem for $e' = 0.2$, $\mu \approx 0.003$ ($m = 0$, $m' = 3m_J$).

When the Trojan planet has non-zero mass, the stability region is still quite extended. Figs. 6 and 7 show the cases when $m = 1m_E$ and $10m_E$ (in both cases $e' = 0$, $m' = 1m_J$). A comparison with Fig. 1 ($m = 0$, $e' = 0$) reveals that the size of the stability region is about the same for Trojan planets of several Earth-masses as for negligible mass.

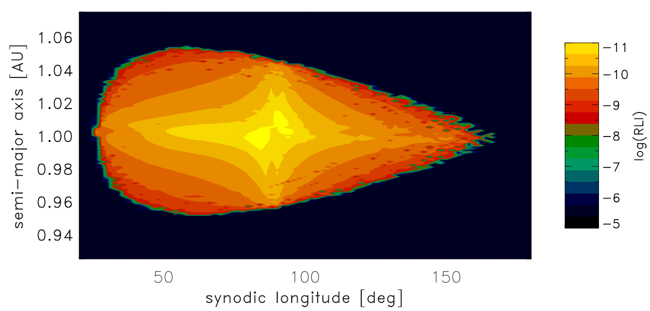

Figure 6. Structure of the stability region around L_4 in the three-body problem for $m = 1m_E$, $e' = 0$, $m' = 1m_J$.

We determined the stability regions around L_4 in the planar three-body problem for the combinations of the masses: $m = 1, 2, 3, 10, 100m_E$, and $1M_J$, $m' = 1, 2, 3, 4, 5, 6, 7m_J$, and initial eccenticity of the giant planet $e' = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30$. For a given pair of m' and e' the size of the stability region does not change much with the increase of m. The changes are larger when m is fixed, and either m' or e' is changed while the other is kept constant. The computations confirm the existence of a stability region around L_4 even for $m = m_J$, when the mass of the giant planet is several Jupiter-masses and its orbit is very eccentric. Fig. 8 shows the stability region for $m = 1m_J$, $e' = 0.3$, and $m' = 1m_J$. Increasing m' at this value of m and e',

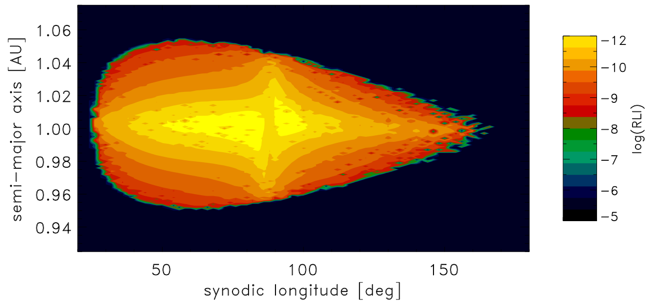

Figure 7. Structure of the stability region around L_4 in the three-body problem for $m = 10m_E$, $e' = 0$, $m' = 1m_J$.

the size of the stability region decreases reaching its minimum at $m' = 5m_J$, after which it grows again, as the computations show.

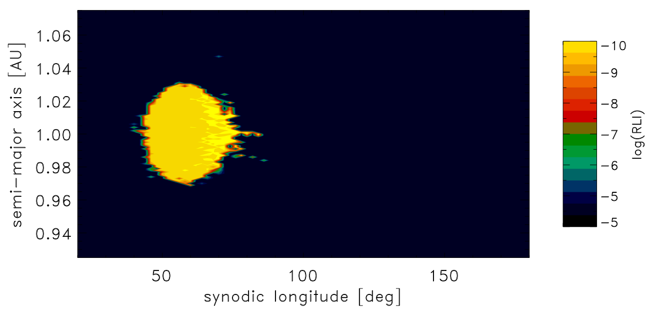

Figure 8. Structure of the stability region around L_4 in the three-body problem for $m = 1m_J$, $e' = 0.3$, $m' = 1m_J$.

4. Size of the stability region

The size of the stability region depends on the masses m, m' and the eccentricity e'. In [2] we determined this dependence for $m = 0$. Continuing that work we studied how the size of the stability region depends also on m. Fig. 9 shows the dependence of the size of the stability region around L_4 on m' and e' for $m = 1m_J$ for 500 periods of the primaries. The figure was obtained as follows.

For a given pair of e' and $\mu = m'/(m_0 + m')$ we put the Trojan planet in the point L_4 with zero relative initial velocity and checked if it stays there or performs librational motion around L_4 for 500 periods of the primaries. (Certainly, the time interval in this kind of investigations is crucial, we took this value as a compromise. The general features of the stability structure appear during this time.) Then we moved the Trojan planet a little away from L_4 along a line going throgh L_4 perpendicular to the line of the primaries. We checked again the librational motion of the Trojan planet. Proceeding in this way we determined the largest distance ε from L_4 (perpendicular to the line of the primaries) at which the Trojan planet starting with zero relative initial velocity still performs librational motion around L_4 and does not cross the line of the primaries. We defined the stability region as the largest possible libration region. Changing e' and μ on a fine grid, we determined for each pair of (e', μ) the largest ε (in the unit of the distance of the primaries) corresponding to the largest libration region. For the sake of better visualization Fig. 9 shows the values of $1/\log(\varepsilon)$ instead of ϵ on a black and white scale. The light region above the V-shaped curve corresponds to instability, libration is possible below this curve. Darker regions correspond to larger librational regions. It can be seen that the size distribution of the stability regions shows a complex structure. The size is the largest when both e' and μ are small ($e' < 0.1$, $\mu < 0.01$). This means that in an exoplanetary system with one giant planet of several Jupiter-masses there can be a Trojan planet of one Jupiter-mass. The fine structure of the figure confirms our previous finding that the size of the stability region changes much either fixing e' and varying μ, or vice versa. There is also an extended stability region for small values of e' $(e' < 0.1)$ between $\mu = 0.014 - 0.02$. This was also found by Lohinger and Dvorak [4]. The unstable regions below $\mu = 0.014$ and at $\mu = 0.023$ correspond to the resonances 3:1 and 2:1 between the frequencies of libration around L_4. The finger-like structure on the left side of the figure may be related to higher order resonances.

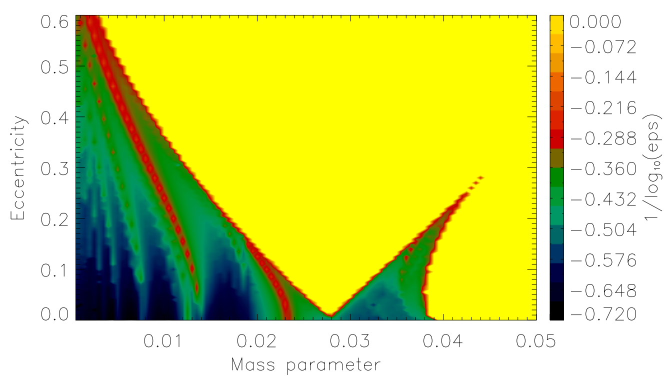

Figure 9. Size of the stability region around L_4 in the planar three-body problem for a Trojan planet of mass $m = 1m_J$ depending on the ecccentricity e' and mass parameter $\mu = m'/(m_0 + m')$ of the giant planet.

Acknowledgments

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References

[1] Dvorak, R., Pilat-Lohinger, E., Schwarz, R., Freistetter, F.: 2004, *Astron. & Astrophys.* **426**, L37

[2] Érdi, B., Sándor, Zs.: 2005, *Celest. Mech. & Dyn. Astron.* **92**, 113

[3] Laughlin, G., Chambers, J. E.: 2002, *Astron. J.* **124**, 592

[4] Lohinger, E., Dvorak, R.: 1993, *Astron.& Astrophys.* **280**, 683

[5] Nauenberg, M.: 2002, *Astron J.* **124**, 2332

[6] Menou, K., Tabachnik, S.: 2003, *Astrophys. J.* **583**, 473

[7] Sándor, Zs., Érdi, B., Efthymiopoulos, C.: 2000, *Celest. Mech. & Dyn. Astron.* **78**, 113

[8] Sándor, Zs., Érdi, B., Széll, A., Funk, B.: 2004, *Celest. Mech. & Dyn. Astron.* **90**, 127