Orbital stability of fictitious Trojans in extrasolar planetary systems

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Introduction

Trojan orbits exist in our Solar system around the Lagrangian points L4 and L5 of Jupiter and – as we know from recent discoveries – also around Neptune. These orbits in the vicinity of the equilibrium points have two libration periods, which depend on the mass ratio of the primary bodies as does the stability of these points (for a mass ratio larger than 1/25 they are unstable even in the circular restricted three body problem Fig. 1).

The growing number of discoveries of massive planets like Jupiter in extrasolar planetary systems (=EPS) opens up the question of the existence of terrestrial planets in these systems. One possibility is that they may have orbits in the 1:1 resonance with a gas giant, which means that they would be Trojan Terrestrial Planets (TTP). Besides the numerical studies for individual EPS, we did some new investigations devoted to establish stability regions around these points in the 3 dimensional elliptic restricted three-body problem (3D-ER3BP).

In our research we took into account possible inclinations of the orbits of the TTPs and studied on one hand the stability of the equilibrium points themselves and on the other hand also the largeness of the stability region depending on the inclination.

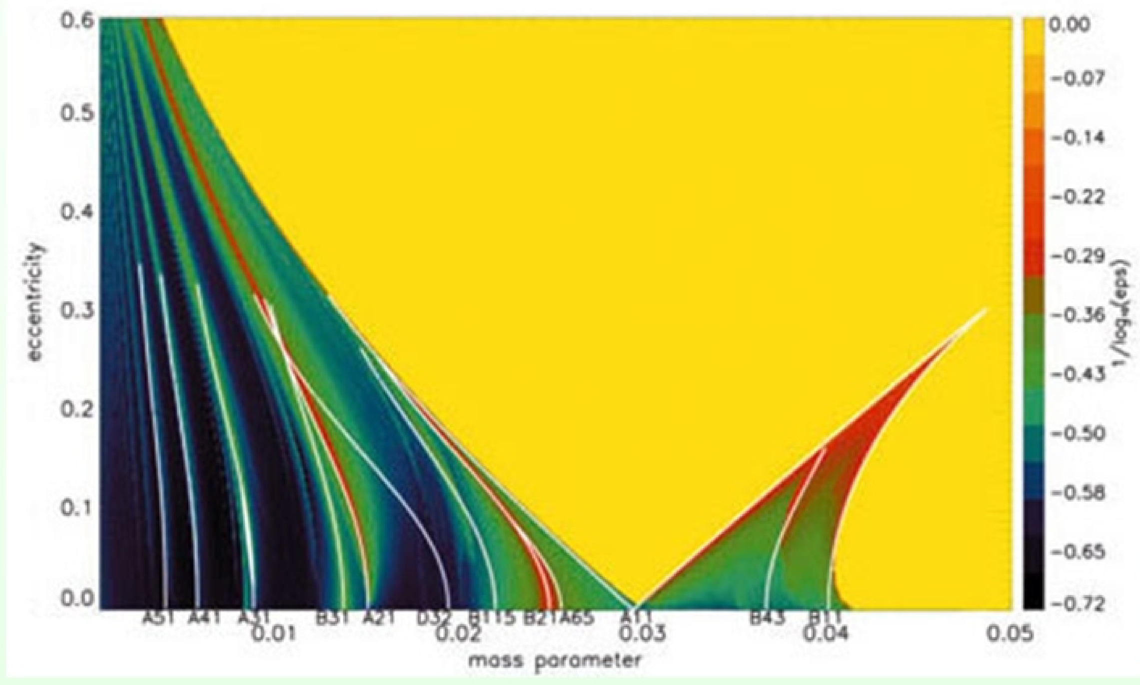

Figure 1: The stability map for the planar case depending on the mass ratio μ and the eccentricity of the Trojan planet. The numerical integration were done for $5x10^4$ periods of the primaries. For any pair of μ and e, we determined the largest distance from L4, denoted by ε, along the line of L4 and the larger primary, until the orbit of the test planet starting from points of this line remains stable in the above sense (Érdi et al. 2007).

Dynamical Model:

Because of relatively large eccentricities of the orbits of known planets in EPS the appropriate this dynamical model chosen for our research is the ER3BP. No analytical results are available for problem – therefore we decided to undertake a numerical study.

Numerical Setup and Method of investigation:

We put the gas giant (GG) in its periastron position and started to integrate the orbit of a massless body put exactly in the Lagrangian point (a=1[AU], ω=60°). We varied the eccentricity e of the primaries' orbit from 0 to 0.3 with a gridsize Δe=0.0075. (The initial eccentricity of the 'Trojan' was always the same as the eccentricity of the GG; the inclination 0° < i < 80° (Δi = 2°). Additionally we investigated the stability for different mass ratios μ of the primary bodies from 0.001 to 0.015 with a gridsize $\Delta\mu$=0.001. For the analysis of the stability we checked the maximum eccentricity during the integration. Whenever it achieved a value larger than e=0.5 the point was classified as unstable. In addition we also computed the Delaunay element H=sqrt(a.(1-e²))cos(i) and checked its time variation.

Results on the orbital elements (Fig. 3):

A) 0.001<μ<0.004: up to i = 60° and e=0.3 L4 is stable, for higher inclinations L4 is unstable.

B) μ=0.005: a finger like unstable structure appears in the stable region for 10°<i<35°and e>0.2.

C) μ=0.008: the former unstable structure is shifted towards smaller eccentricities and larger inclinations, growing in size.

D) μ=0.010: The unstable finger like structure is shifted towards the large unstable region for i>60°; A new unstable region appears for large eccentricities and small inclinations.

E) μ=0.013: this unstable region is growing in size and shifted to smaller eccentricities; the finger like structure disappears in the large unstable zone at i>60°.

F) μ=0.015: Only a stable region of triangular form with a basis between 0<i<60 and a top at i=50° and e=25 is left over.

How can we explain this growing instability?

In Fig. 1 for the planar ER3BP we can see a similar structure with several tongues inside the stable region produced by high order resonances (Érdi et al. 2007) . We cannot yet identify these fingers in the respective plots (Fig. 3) but an analytical investigation has been started (Marchal, 2007).

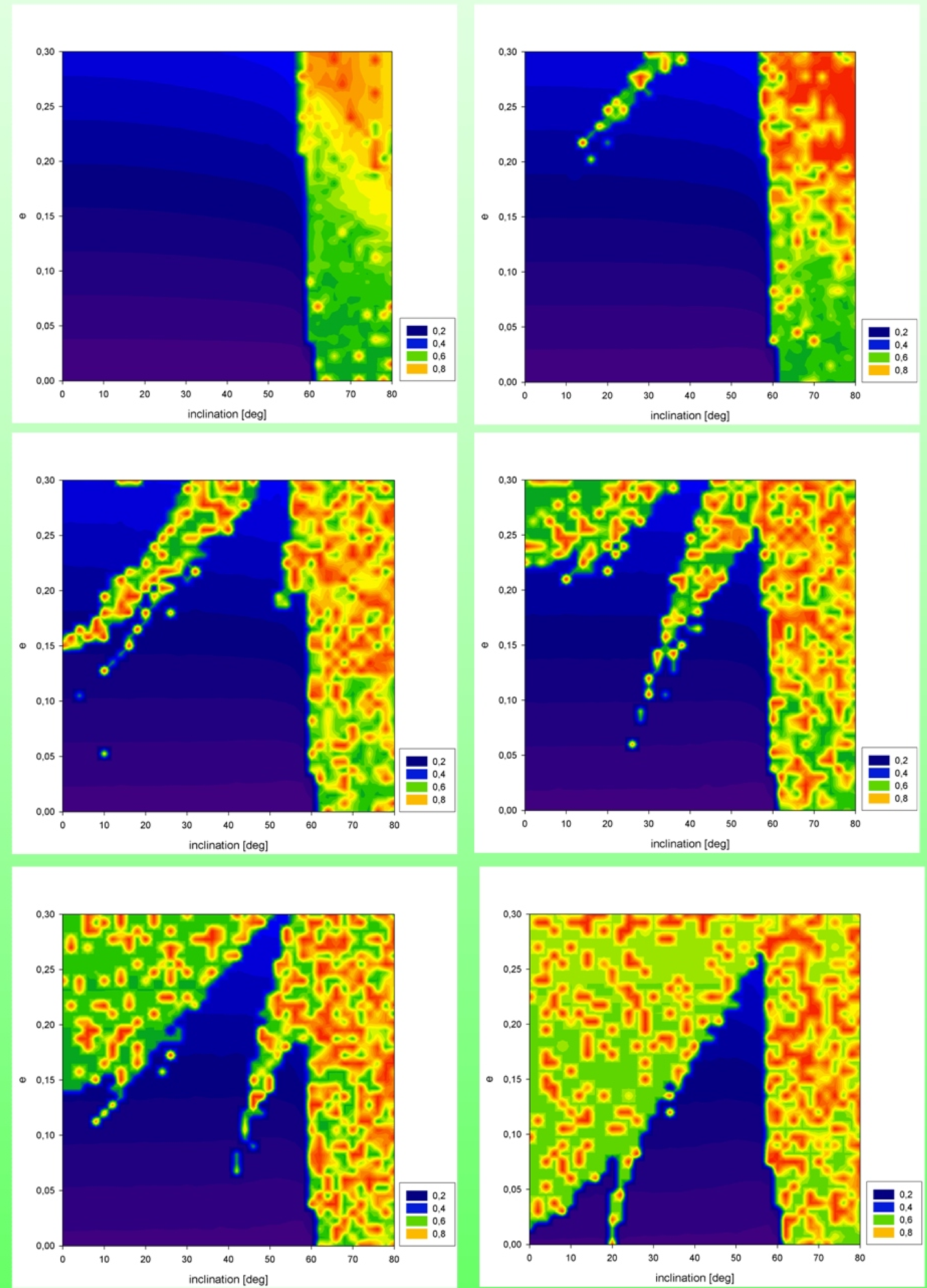

Figure 3: In the above stability plots (horizontal-axes: inclination, vertical-axes: orbital eccentricity of the primaries) we marked the value of the maximum eccentricity of the test body with different colours (from blue to red, stable to unstable)

The stability maps are for 6 different mass ratios: The upper two graphs: μ = 0.001 (left graph) and μ = 0.005 (right one); the middle two: μ = 0.008 (left graph) and μ = 0.010 (right one); the lower two: μ = 0.013 (left graph) and μ = 0.015 (right one).

The intergrations were done for 10^6 years.

Proper element space:

The proper orbital elements of an object are approximate constants of motion that remain practically unchanged over an astronomically long timescale. Érdi (1988) described the long periodic variations in the expansion of the semimajor axis a and the relative mean longitude $\lambda - \lambda^*$ for a Trojan asteroid in the following way (* refers to the elements of the planet with the mass ratio μ): $a - a^* = df \sin\theta + O(df)^2$; $\sigma = \lambda - \lambda^* = \pm\pi/3 + Df \cos\theta + O(Df)^2$, where df (in AU) and Df (in radians) are the amplitudes of libration in semimajor axis and the critical argument , respectively, which are related as df = $\sqrt{3\mu}\cdot$a*Df~0.3741Df.

Results:

We could confirm the restults of the largeness of the stable zone around L4/L5 for the planar case like in e.g. (Marzari & Scholl 2002 and Levison et. al. 1997). Additionally we found that for low inclinations (up to i=10°) the stability maps look similar, but between 10° and 20° the stable region grows with the increase of the inclination. More investigations have to be done for higher inclinations.

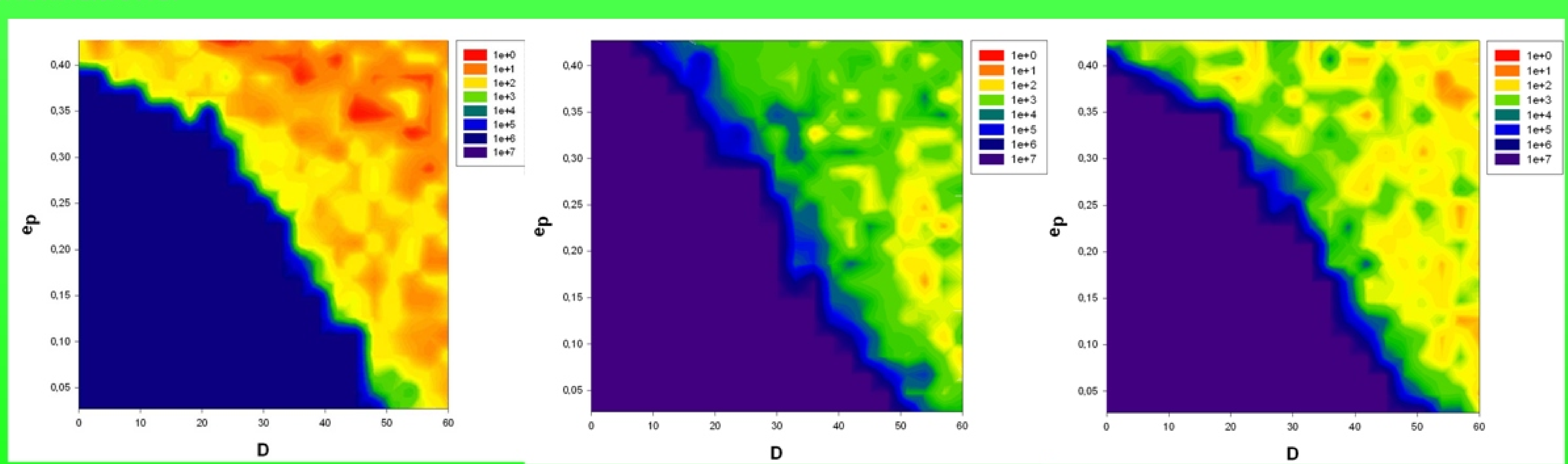

Figure 4: The stability regions around the equilibrium points in the proper element space. (horizontal axes: amplitude of libration, vertical axes: proper eccentricity of the Trojan planet). The colour code is the same as in Fig.3. The left graph present the stability-map for i=0°, the middle one for i=10° and the right one for i=20°.

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