The Discrete Virial Theorem

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The virial theorem is an important tool from classical mechanics, with many applications throughout the physical sciences. In this talk we present new orbital virial theorems for both Hamiltonian flows and symplectic maps. In particular we consider natural Hamiltonians of the form $H = T + U$, where T is a positive definite quadratic form in the momenta and $U(\mathbf{q})$ is a smooth potential. For bounded orbits the classical virial theorem relates the time-averaged orbital kinetic and potential energies in arbitrary dimension. A strong and a weak form of this theorem are proven for both flows and natural maps. While the Birkhoff Ergodic Theorem guarantees the existence of the relevant time averages for both regular and chaotic orbits, the convergence is very rapid for the former and extremely slow for the latter. This circumstance leads to a simple and efficient measure of chaoticity, which we call "meander." The results are applied to several examples of current interest, including the Hénon-Heiles system and weak chaos in the standard map.