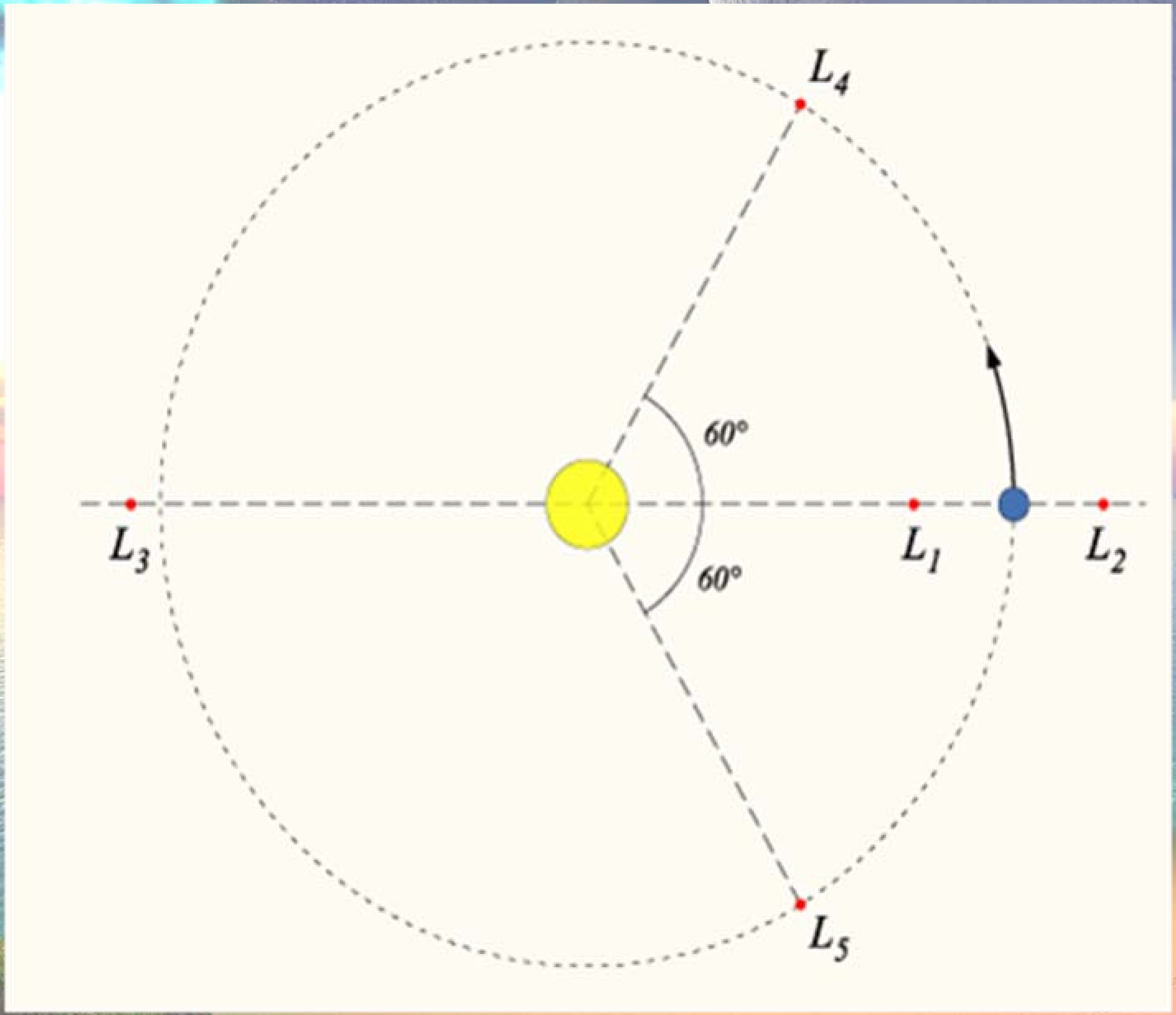
The Three Trojan Problem

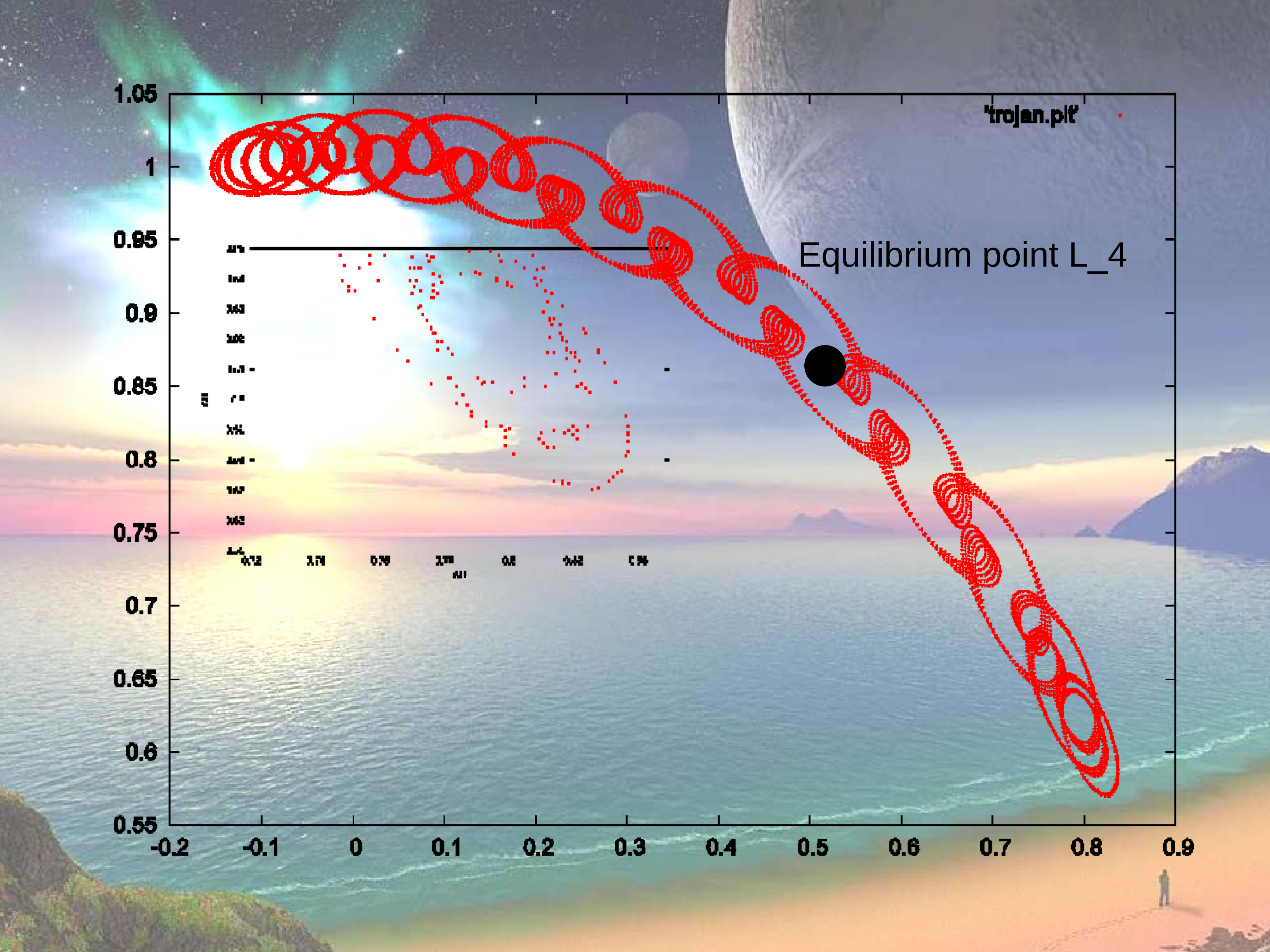
5th Austrio-Hungarian Workshop

April, 9th – April, 10th

Vienna Observatory

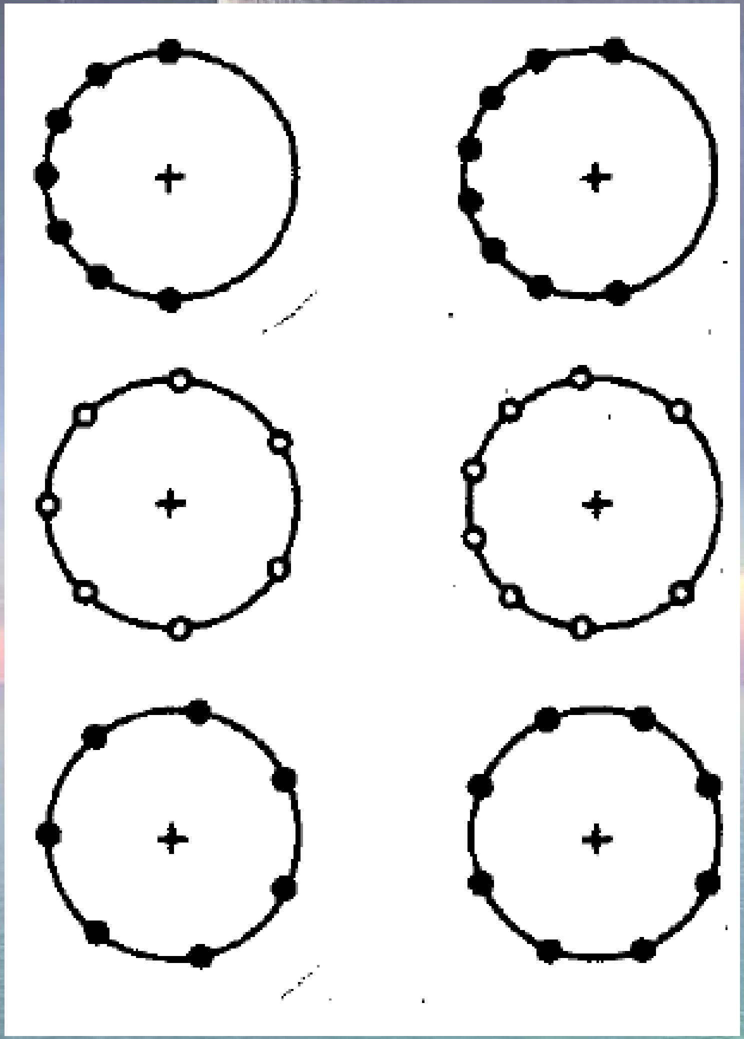
The five Lagrange points SUN + JUPITER





'trojan.plt'
Equilibrium point L_4
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

What happens when we have not only 2 equilibrium points but several- a whole ring of celestial bodies around a central body?



The dynamics of coorbital satellite systems

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Summary. The dynamical behavior of N coorbital satellites, moving with the same average mean motion around a primary has been studied both analytically and in terms of numerical integrations for $2 \leq N \leq 9$ satellites. Simplified dynamical equations have been used to determine the different stationary configurations and their local stability against infinitesimal perturbations. The motion is reduced to angular separations between satellites and is accurate to the first order in satellite to primary mass ratio. The ring of equally spaced identical satellites is found to be locally unstable for $N \leq 6$, while for $2 \leq N \leq 8$ there exists another, stable compact stationary configuration, with separations $\leq 60°$ between adjacent satellites. For $N \geq 7$ the equally spaced configuration becomes locally stable, and for $N \geq 9$ it is the only stationary configuration. Exact integrations

2. Simplified dynamical equations

The equations of motion for N mutually gravitating satellites, moving around a central mass in coplanar orbits, can be written in the form (see e.g. Yoder et al., 1983; Brown and Shook, 1964),

$$\frac{d}{dt}\left(r_i^2 \frac{d\theta_i}{dt}\right) = \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\partial F_{ij}}{\partial \theta_i}, \tag{1}$$

$$\frac{d^2 r_i}{dt^2} - r_i \left(\frac{d\theta_i}{dt}\right)^2 = -\frac{\gamma M}{r_i^2} + \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{\partial F_{ij}}{\partial r_i}, \tag{2}$$

where

$$F_{ij} = \gamma m_j \left\{\frac{1}{\Delta_{ij}} - \frac{r_i}{r_j^2} \cos(\theta_i - \theta_j)\right\}, \tag{3}$$

$$\Delta_{ij}^2 = r_i^2 + r_j^2 - 2 r_i r_j \cos(\theta_i - \theta_j). \tag{4}$$

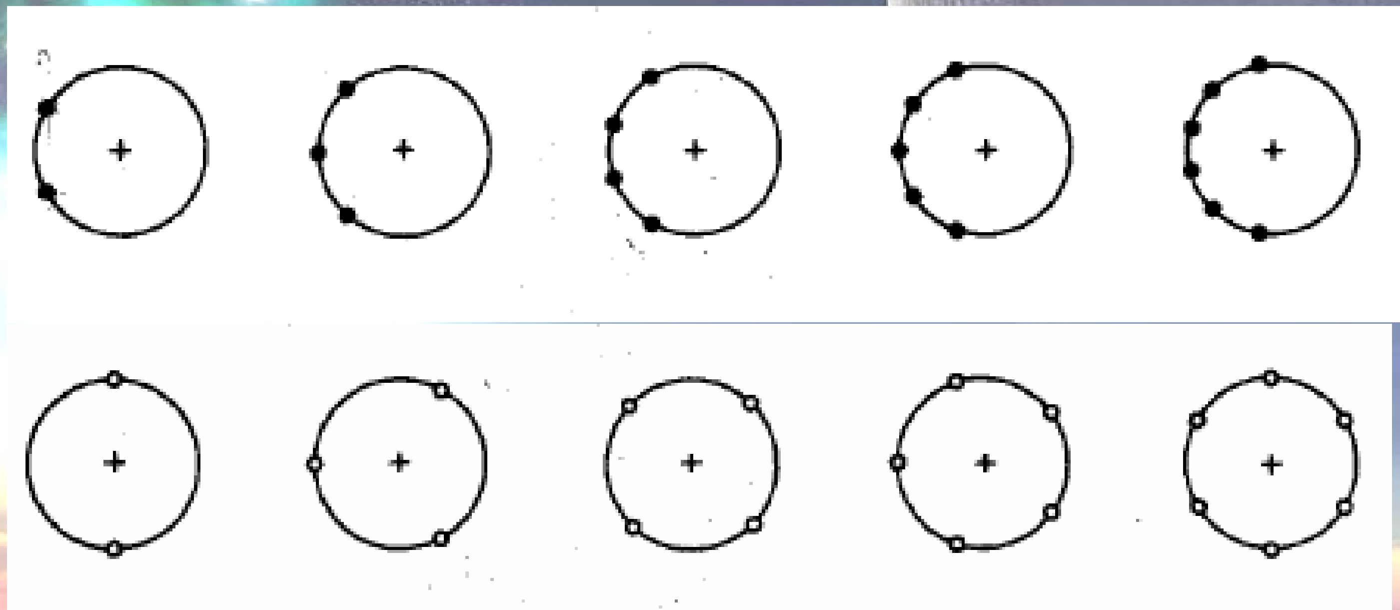
The polar coordinates (r, θ) are referred to the center of mass of the primary, which gives rise to the indirect terms $\sim \cos(\theta_i - \theta_j)$ in the forcing functions F_{ij}. Satellite masses are denoted by m_i, while M stands for the central mass and γ for the gravitational constant.

For $N > 2$ the situation is more complex, due to the large number of dependent variables: the energy condition is not sufficent to determine all the difference angles. However, for $N = 3$ some progress can be made by plotting the mutual gravitational potential as a function of angular separations, $\phi_i = \theta_{i+1} - \theta_i$. For convenience, assume $\theta_1 < \theta_2 < \theta_3$, and define new variables,

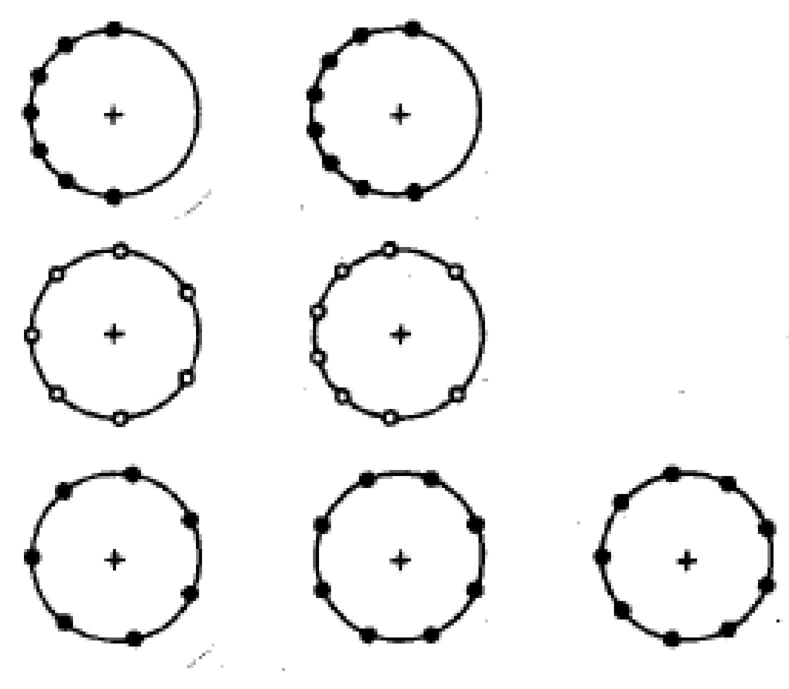
$$\begin{aligned} \alpha &= (\phi_1 + \phi_2)/2 = (\theta_3 - \theta_1)/2\,; &\qquad 0^\circ < \alpha < 180^\circ\,, \\ \beta &= (\phi_1 - \phi_2)/2 = \theta_2 - (\theta_3 + \theta_1)/2\,; &\qquad -\alpha < \beta < \alpha\,, \end{aligned} \tag{12}$$

so that α corresponds to the mean interparticle separation, while β describes the deviation of the middle particle from the equidistant position. Figure 1a shows the potential surface as a func-

2 3 4 5 6



Full cercles –stable orbits



7 8 9

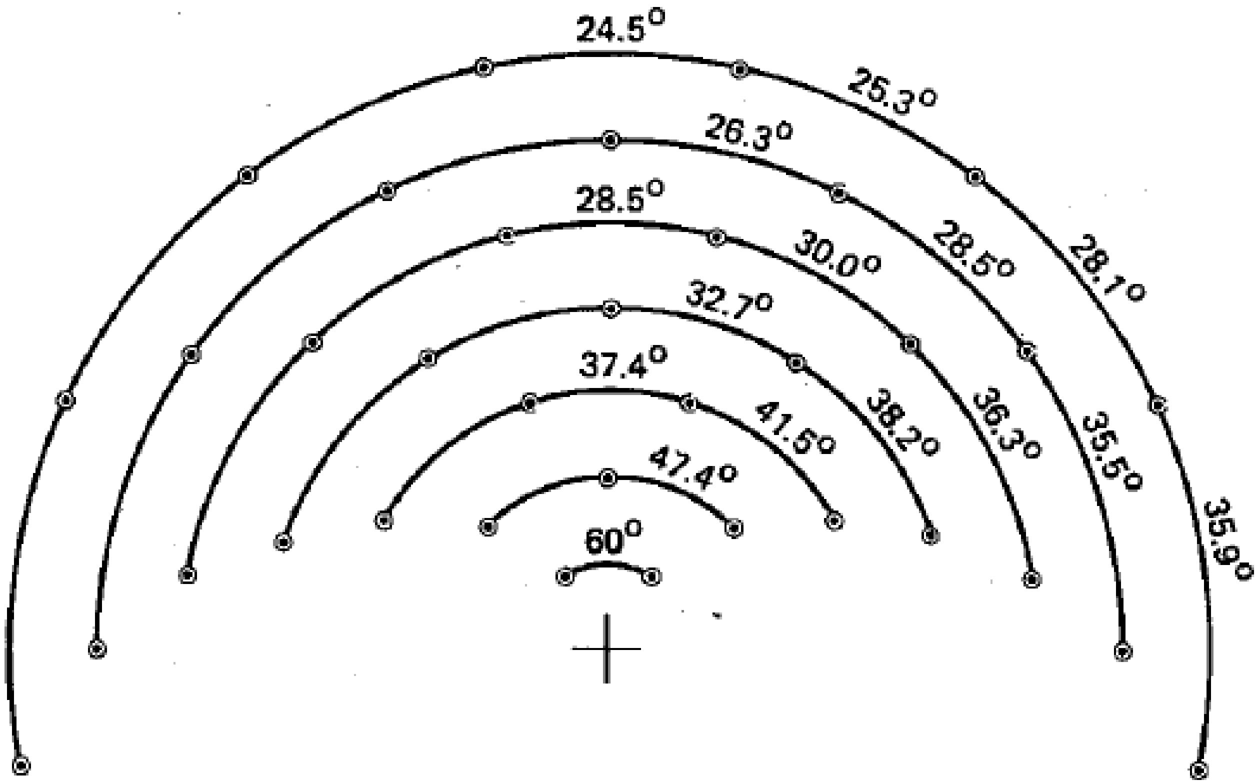
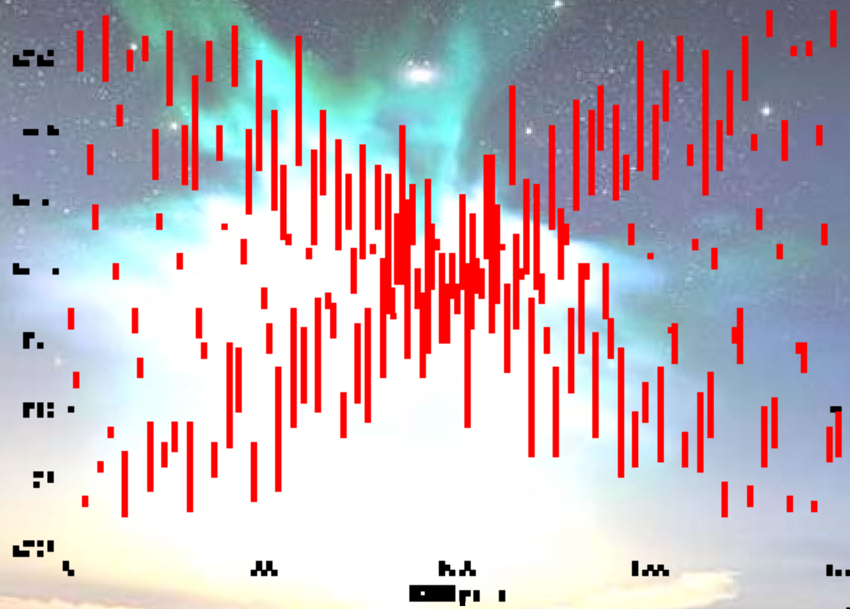


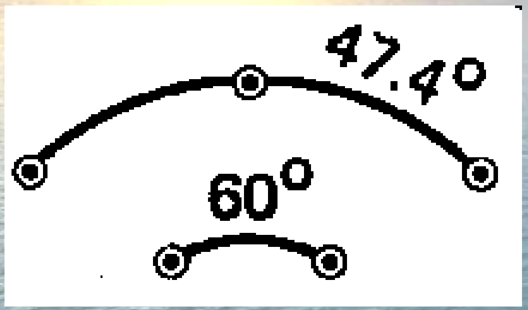
Fig. 3. The angular separations for the stable compact configuration (Type I), for $2 \leq N \leq 8$ satellites

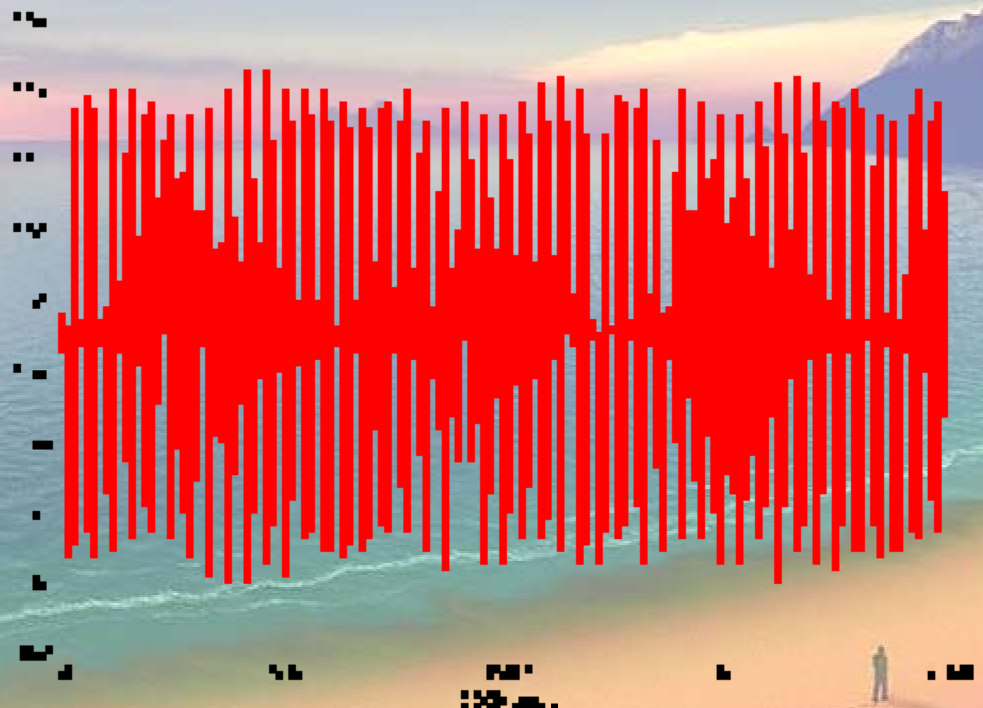


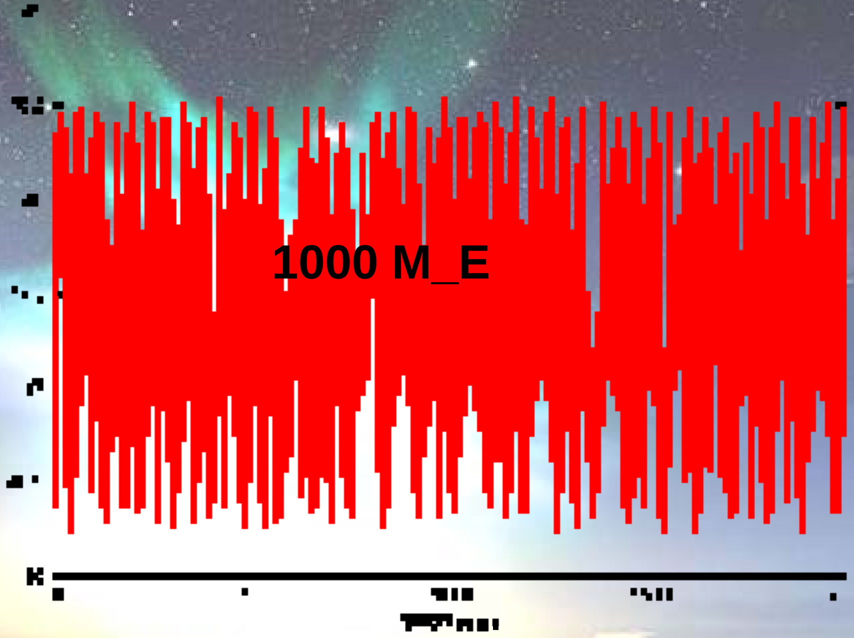
1 m_E

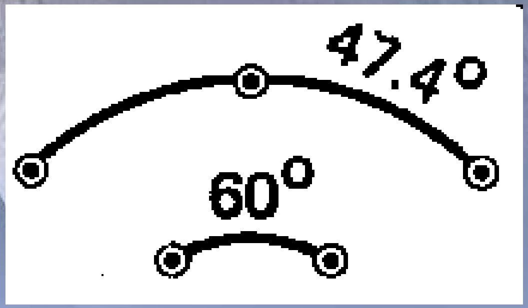
Libration around ,L_4'

100 m_E

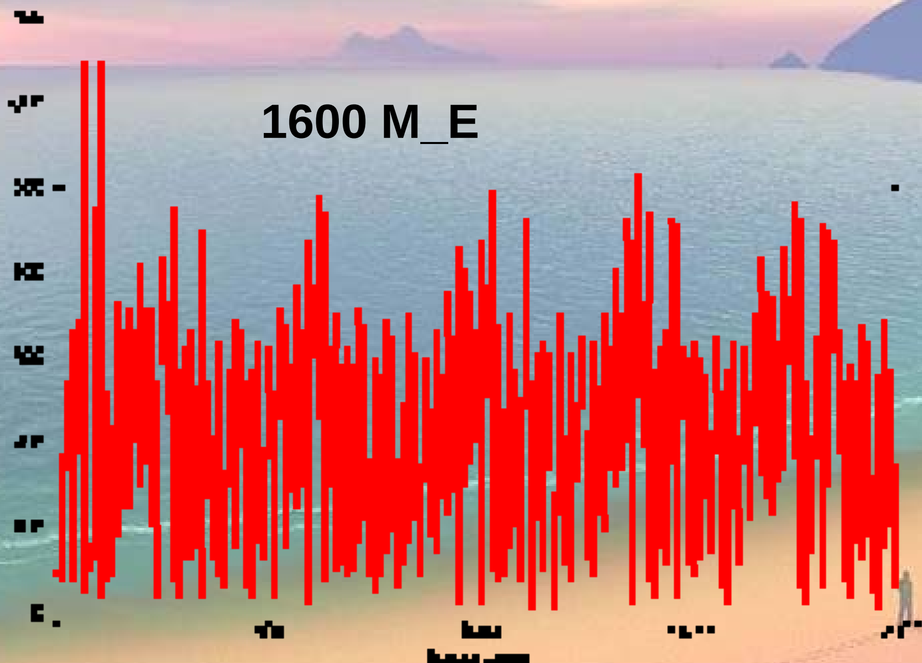




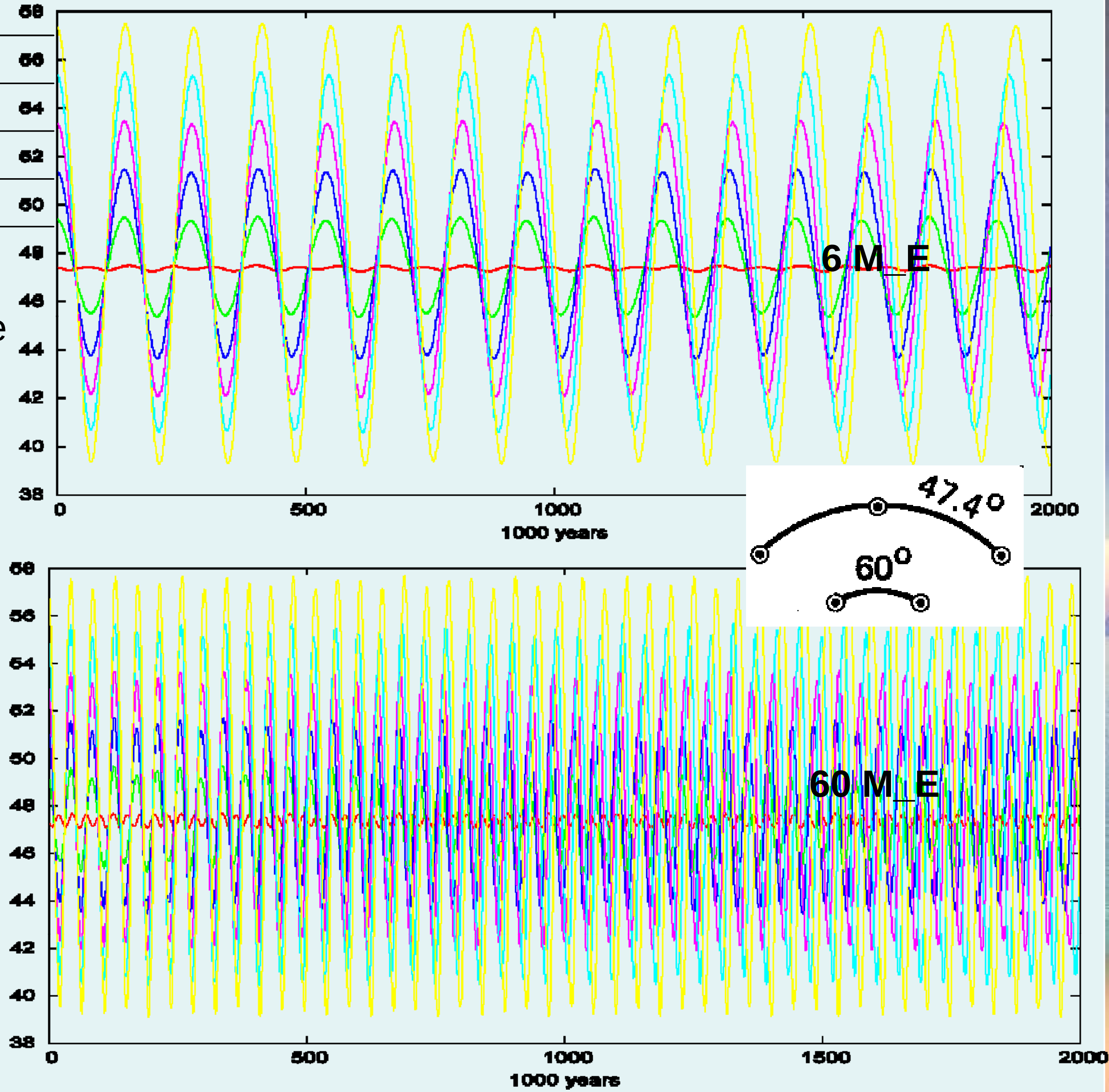




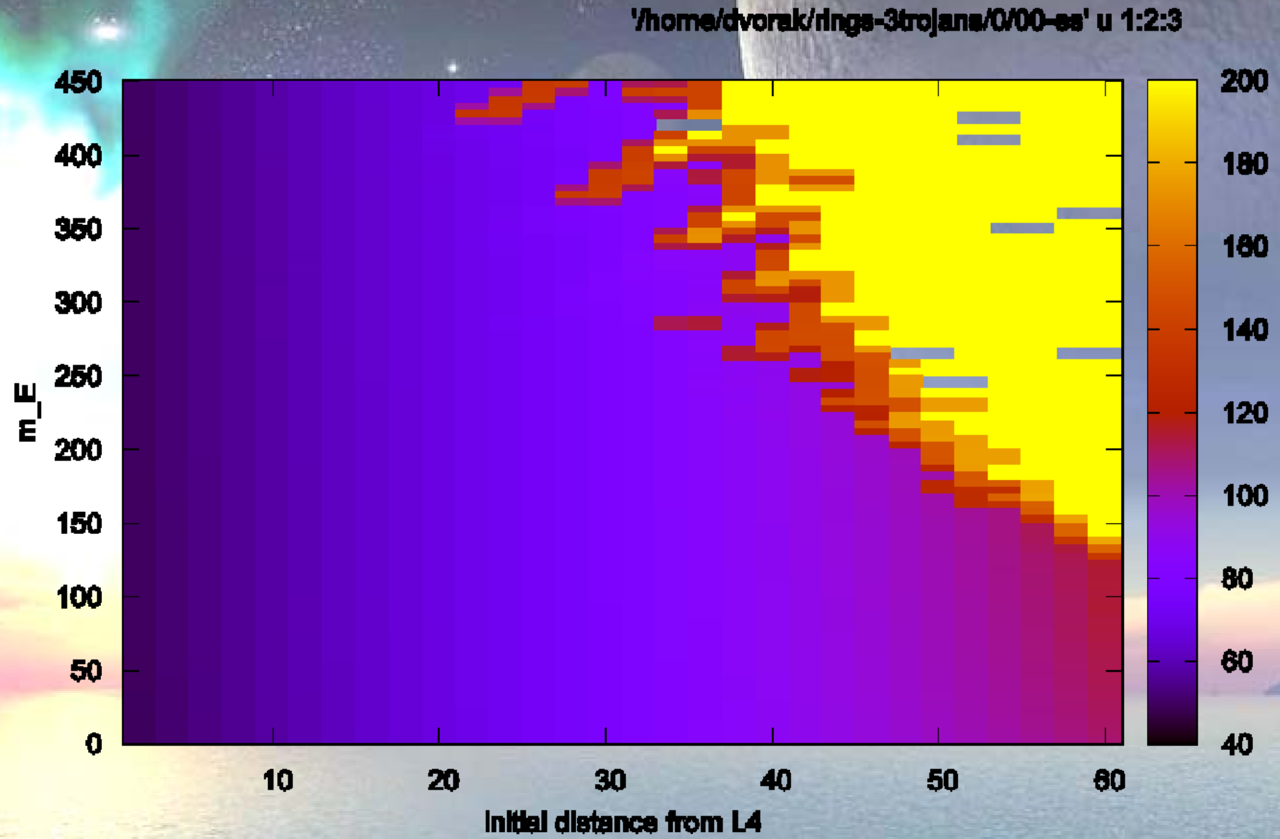
Libration around ‚L_4'

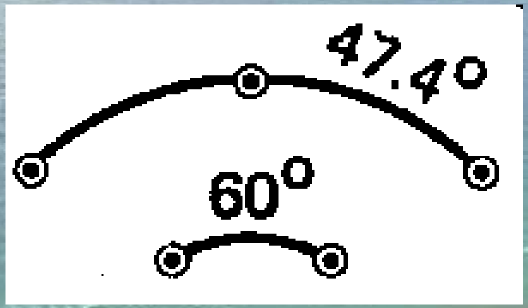


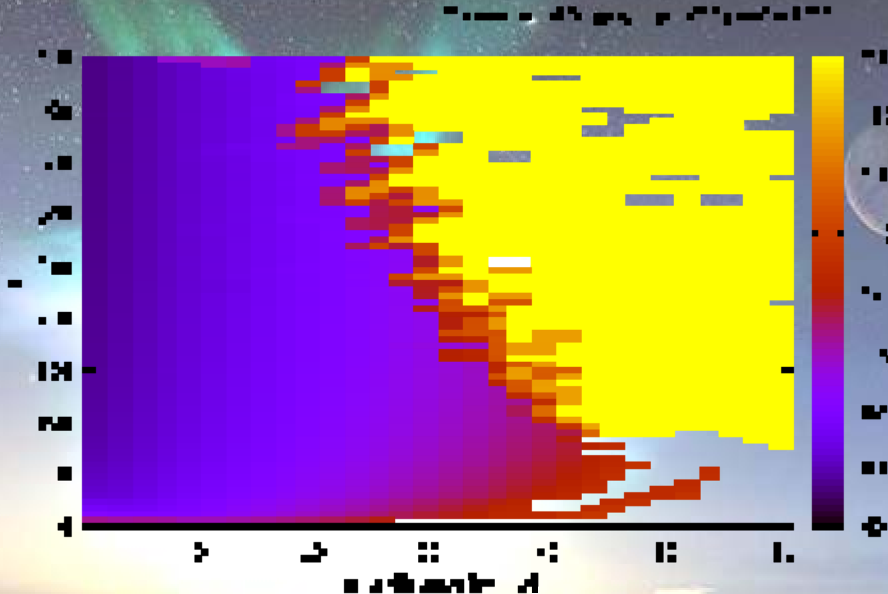
Initial distance from central Trojan



Libration


'/home/dvorak/rings-3trojans/0/00-es' u 1:2:3
m_E
initial distance from L4
450
400
350
300
250
200
150
100
50
0
10
20
30
40
50
60
200
180
160
140
120
100
80
60
40

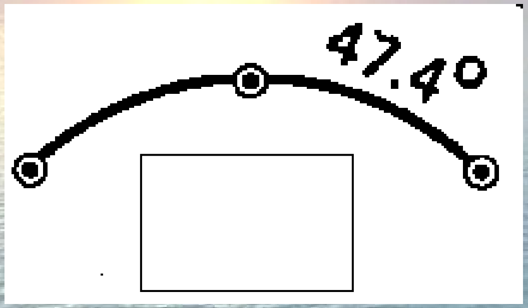

47.4°
60°

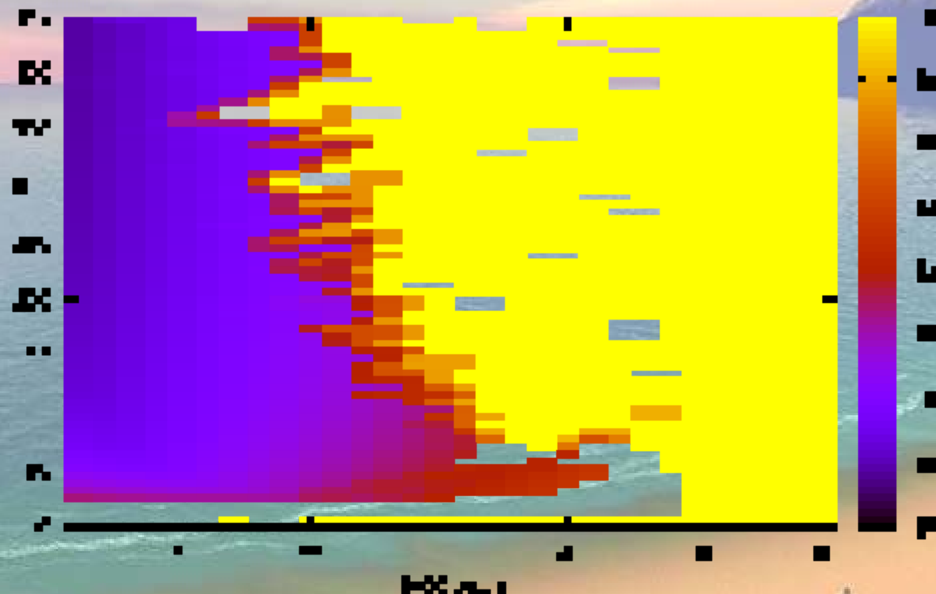


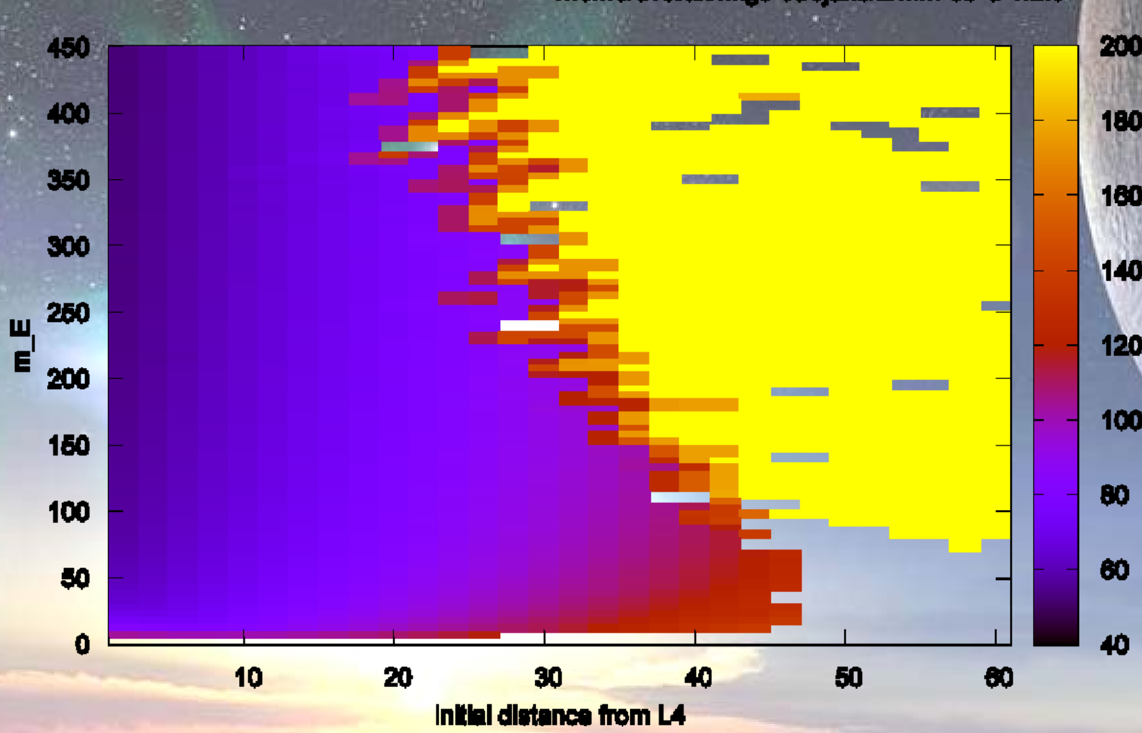
a=a+0.01

a=a+0.02

pp

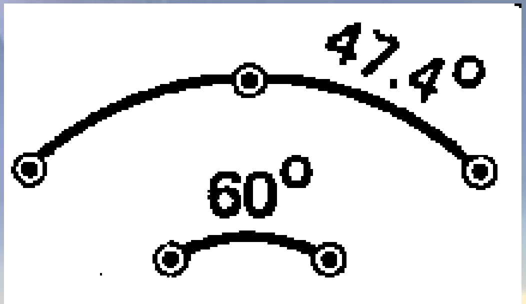






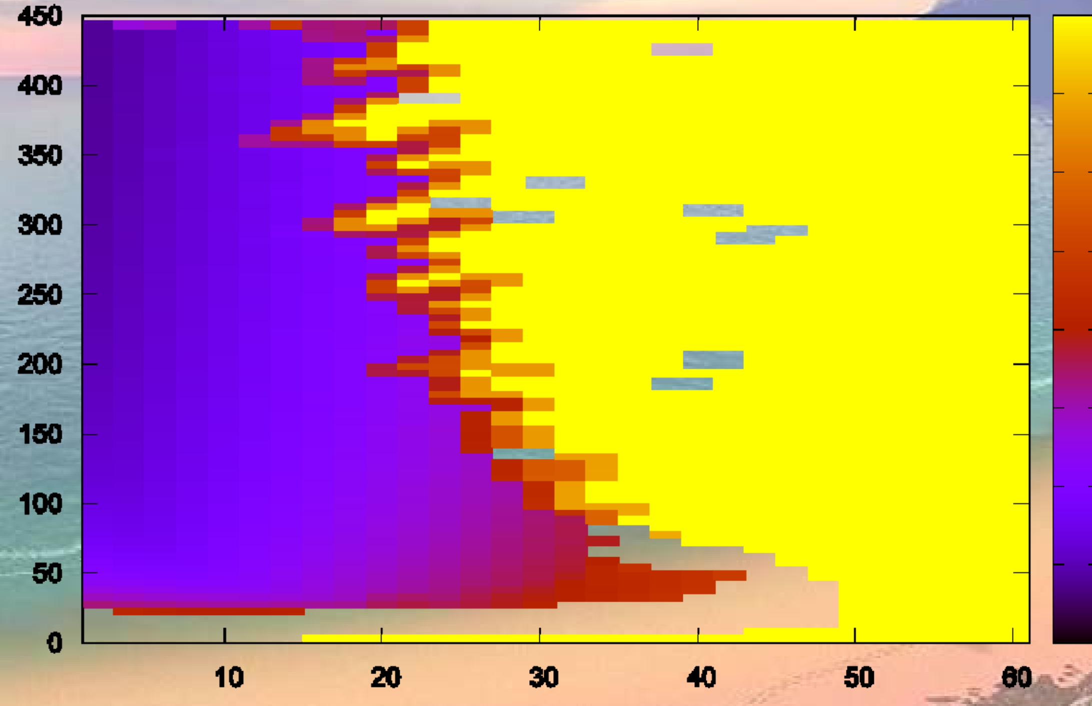
a=a-0.01

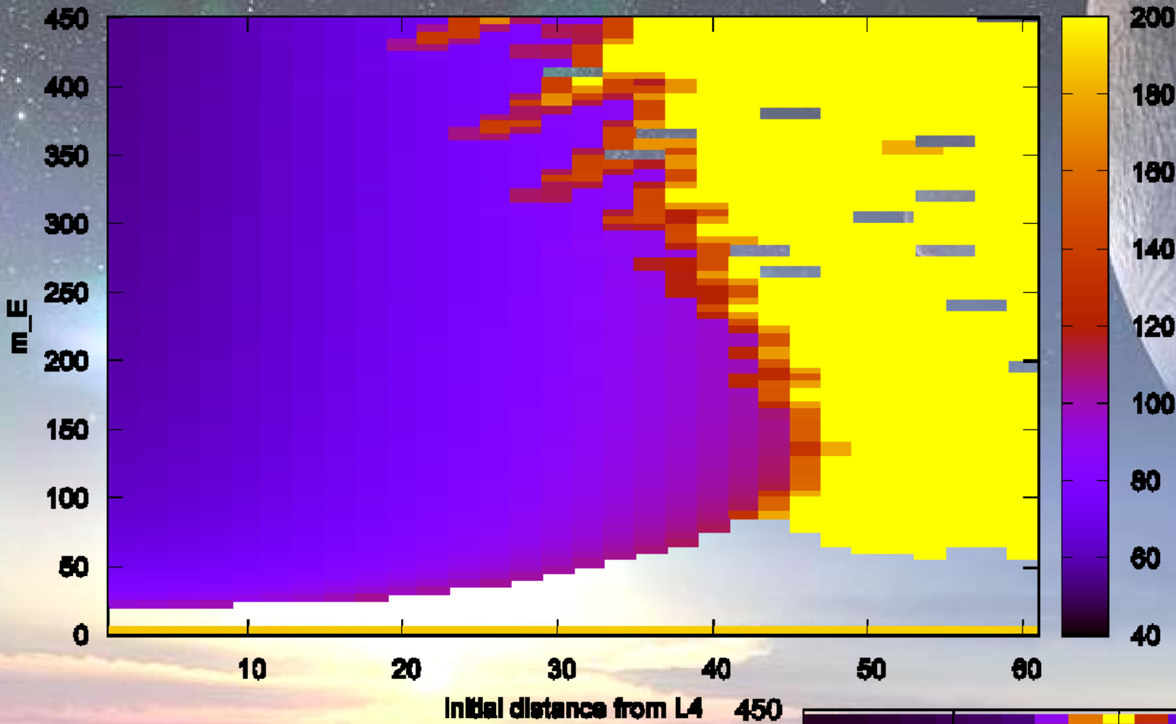
a=a-0.01



a=a-0.02

a=a-0.02



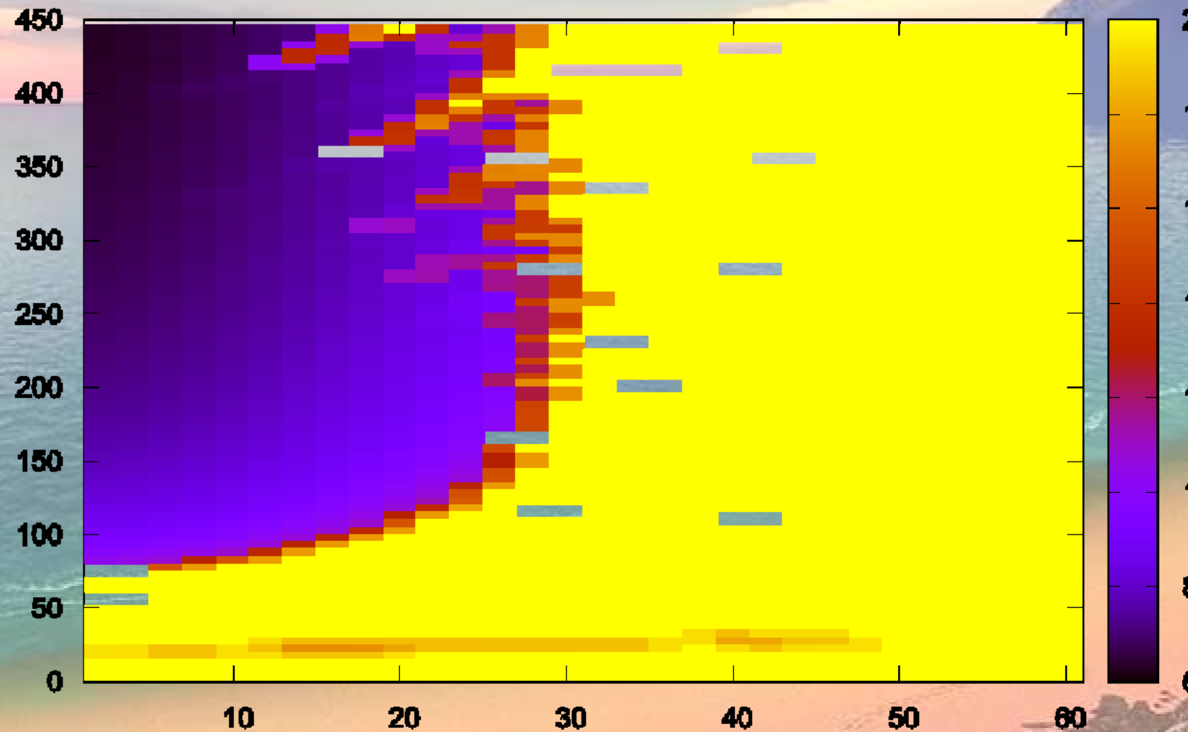


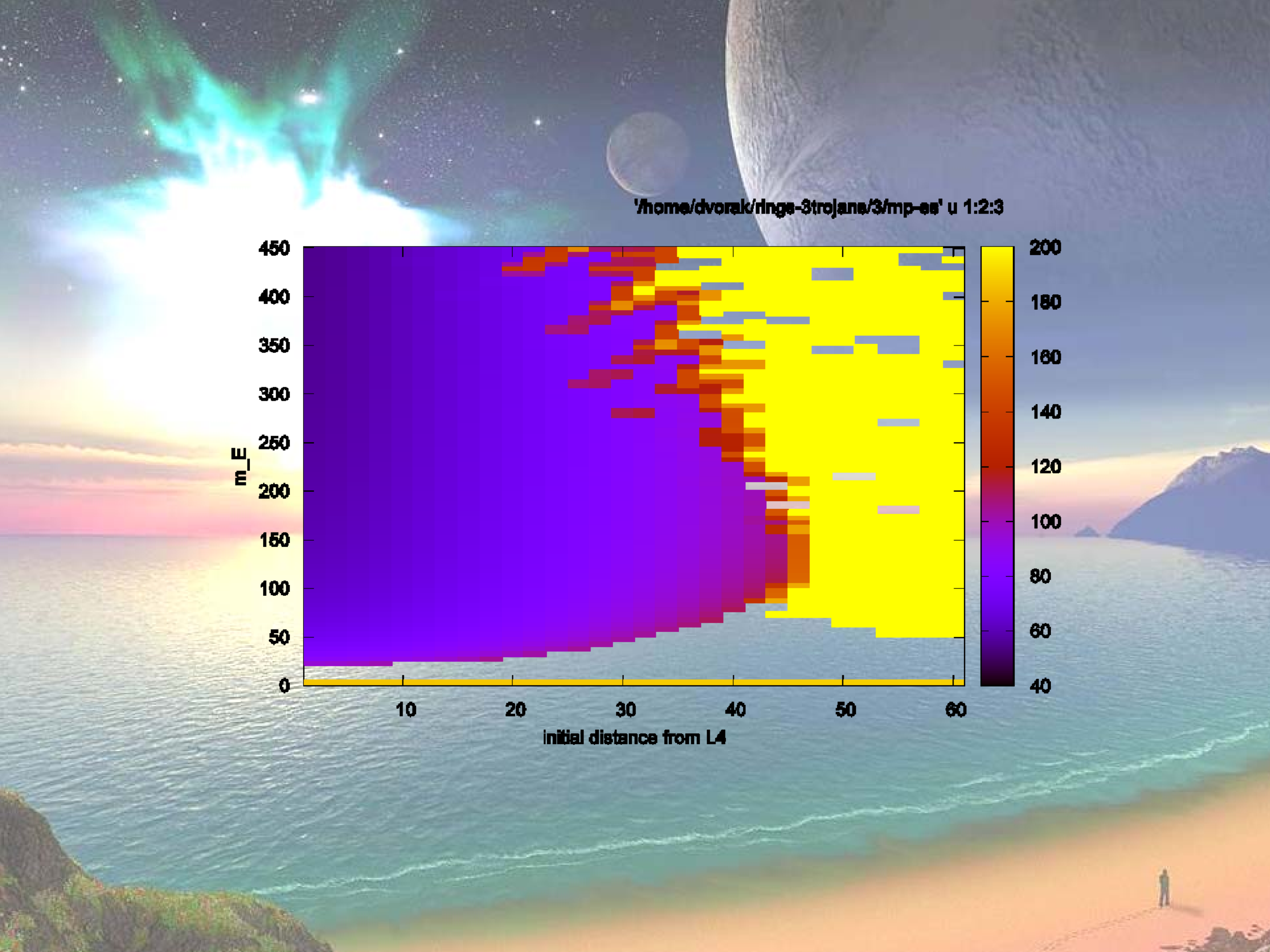
a=a+0.01

a=a-0.01

a=a+0.02

a=a-0.02



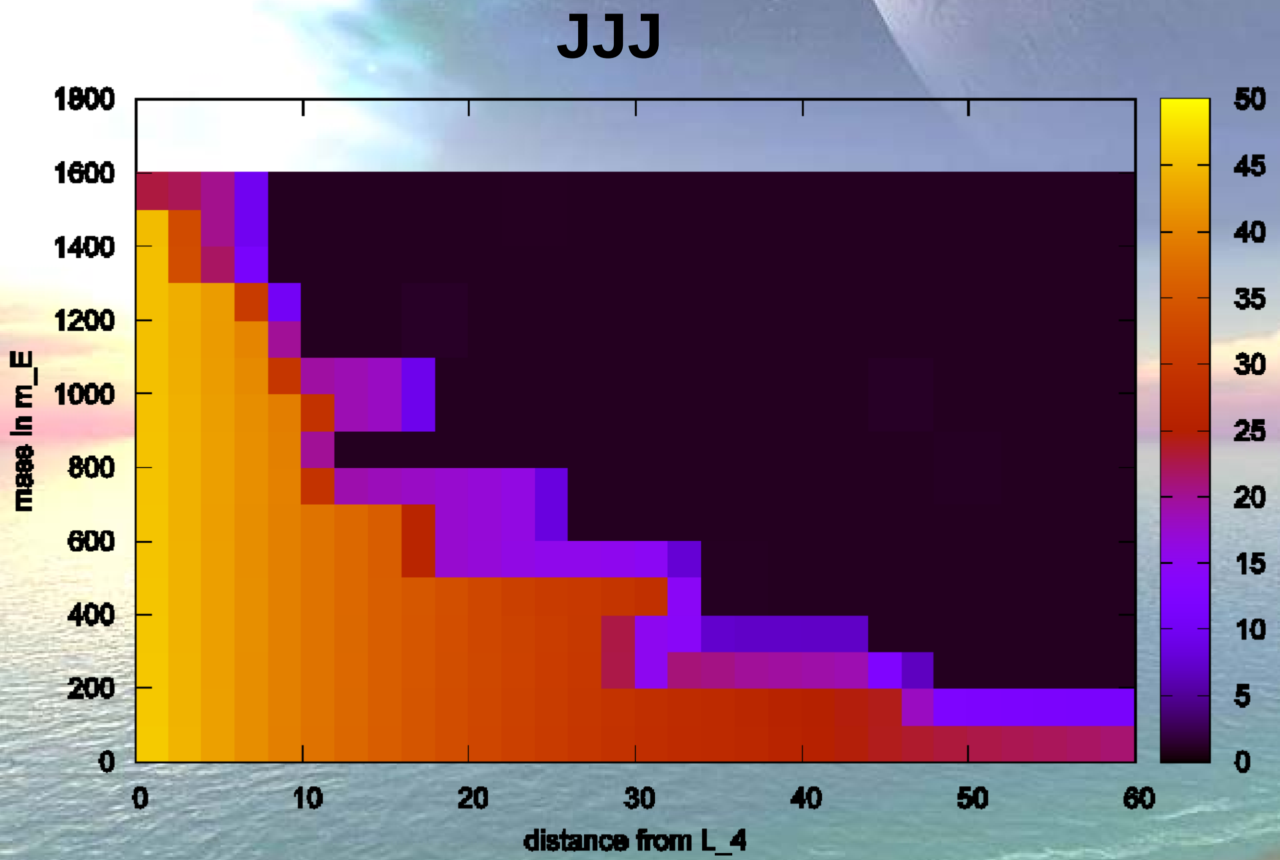


'/home/dvorak/rings-3trojans/3/mp-es' u 1:2:3
m_E
0
50
100
150
200
250
300
350
400
450
10
20
30
40
50
60
initial distance from L4
40
60
80
100
120
140
160
180
200

Three equal masses

Extension to very large masses

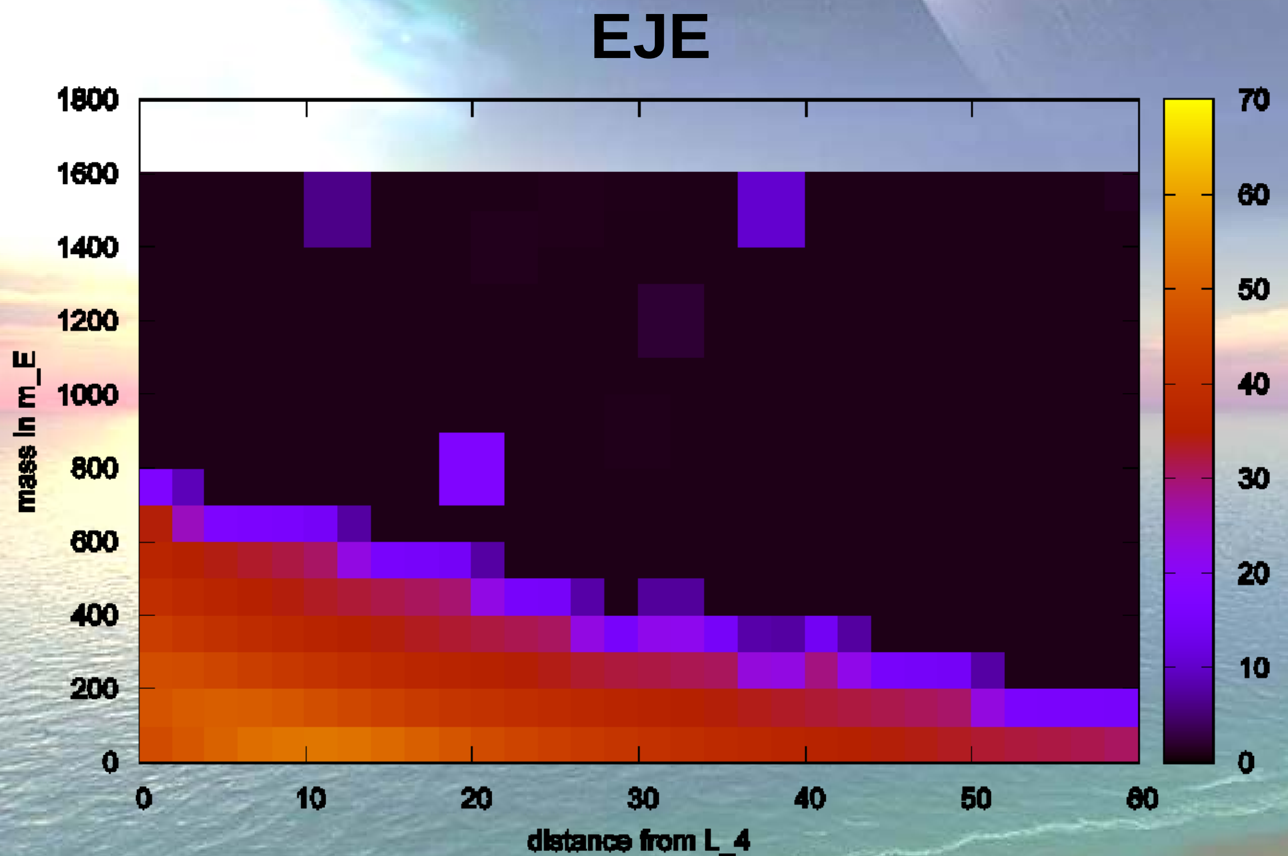
1500 = 5 M_Jup



Two equal masses

Extension to very large masses

1500 = 5 M_Jup



Conclusions

Stable region around equilibrium points

a=a+/-0.02

m<1500 m_E

Libration for L_4 for I<~100deg

Ongoing study:

Mass ratios different

a=a+/-0.0?

Inclinations?