

# Dynamics of possible Uranus Trojans

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# Outline

- 1 Motivation
- 2 Dynamical Model & Methods
- 3 Analysis
- 4 Results
  - Cuts through L4/L5
  - Libration width
  - Long-time integration
  - Frequency analysis





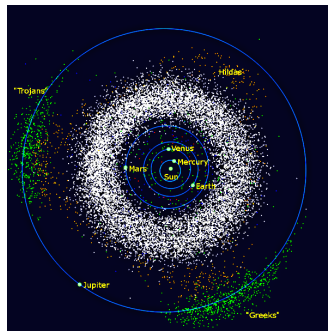
# Trojans in the Solar System

Planets with known Trojan asteroids<sup>a</sup>:

- + Mars:  
 $L_4 = 1, L_5 = 3$
- + Jupiter:  
 $L_4 = 2603, L_5 = 1473$
- Saturn: 0
- Uranus: 0
- + Neptune:  
 $L_4 = 6, L_5 = 0$

<sup>a</sup>source: IAU Minor Planet Center,  
list dated from 2010 Feb. 12,

<http://www.cfa.harvard.edu/iau/lists/Trojans.html>



Jupiter Trojans (source from: Wikipedia)





# Trojans in the Solar System

## Goals for investigation

- 1 Dynamically stable zone for Uranus Trojans?
- Murray & Holman (1999): overlap of MMR in OSS
- Nesvorný & Dones (2002): population depleted by factor 100
- Zhou et al. (2009): Neptune Trojans at high inclinations

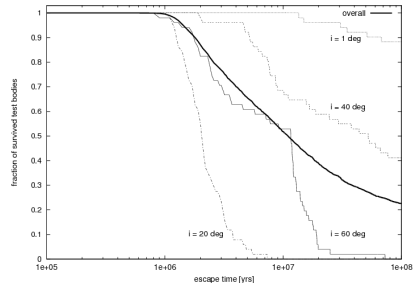




# Trojans in the Solar System

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- 1 Dynamically stable zone for Uranus Trojans?
- 2 Timescale for stability?





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# Trojans in the Solar System

## Goals for investigation

- 1 Dynamically stable zone for Uranus Trojans?
- 2 Timescale for stability?
- 3 Dependence on inclination?
- 4 Perturbations by resonances?





# Dynamical Model

- OSS: Sun – Jupiter – Saturn – Uranus – Neptune
- restricted problem: swarm of massless Trojan asteroids
- Newtonian framework
- equilateral triangle equilibrium points  $L_4$ ,  $L_5$  dynamically symmetric





# Methods

- numerical integration of equations of motion: Lie Integrator
- integration time:  $10^6 - 5 \times 10^9$  years
- grid of initial conditions:
  - changed  $a = a_U \pm m \times 0.007 \text{ AU}$  ( $1 \leq m \leq 50$ )
  - changed  $i_U \leq i \leq 60^\circ$
  - $\omega_T = \omega_U \pm 60^\circ$
- $(e, \Omega, M)$  identical to Uranus
- cuts through equilibrium points (supposed values)





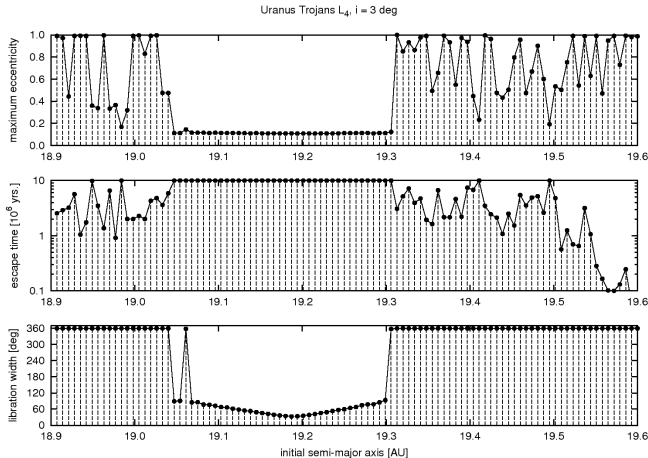
# Tools for analysis

- 1 maximum eccentricity:  $e_{\max}$
- 2 libration width:  $\sigma = \lambda - \lambda_U$
- 3 escape times
- 4 frequency analysis: proper frequencies



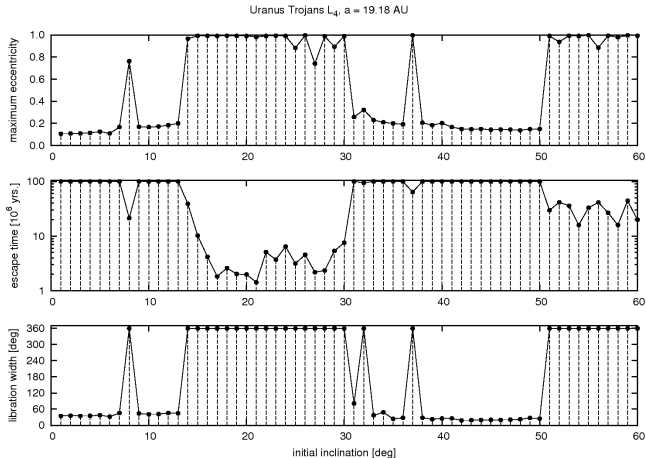


# Stability of $L_4$ cut in semi-major axis



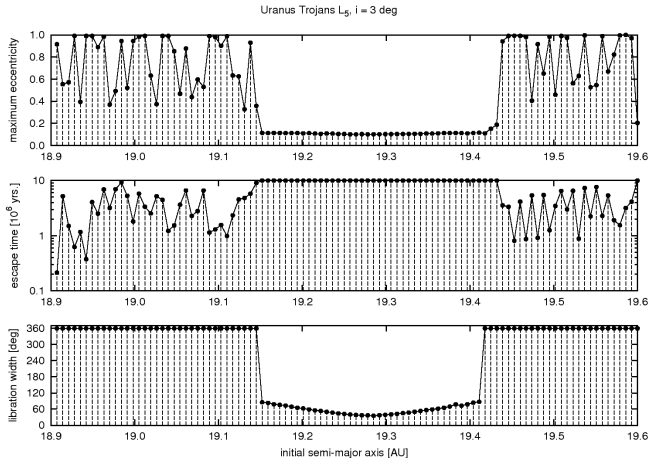


# Stability of $L_4$ cut in inclination



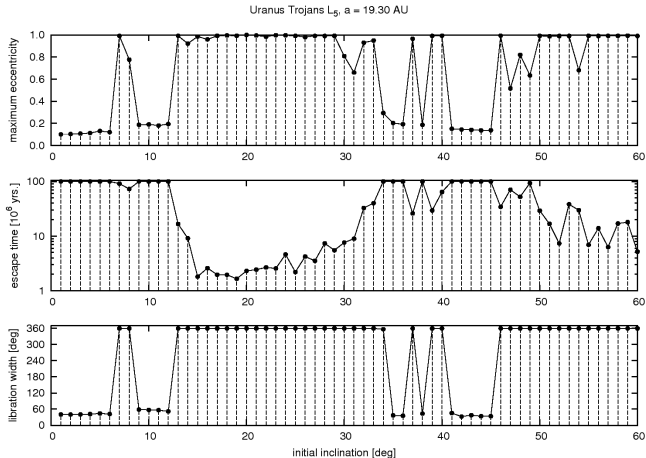


# Stability of $L_5$ cut in semi-major axis



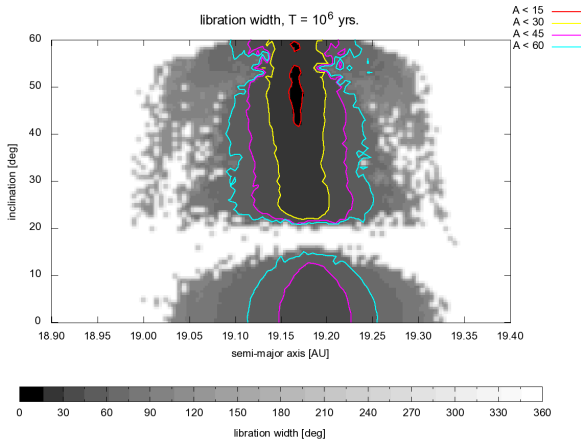


# Stability of $L_5$ cut in inclination



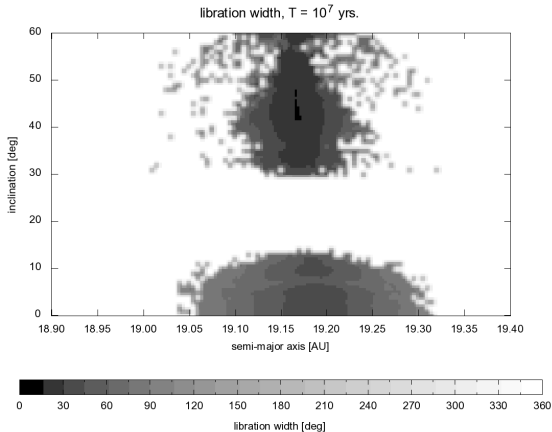


# Libration Width (1/3)



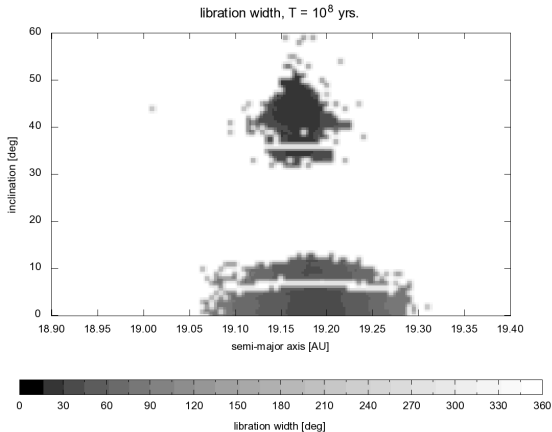


## Libration Width (2/3)





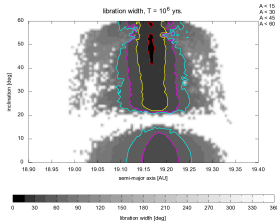
# Libration Width (3/3)



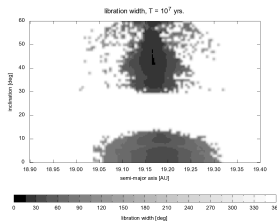


# Comparison of Libration Widths

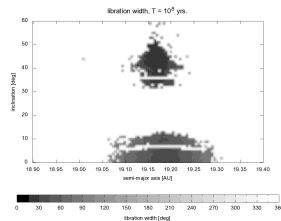
$T = 10^6$  yrs.



$T = 10^7$  yrs.



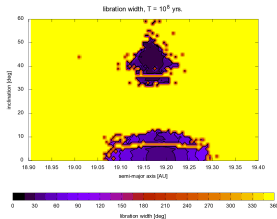
$T = 10^8$  yrs.



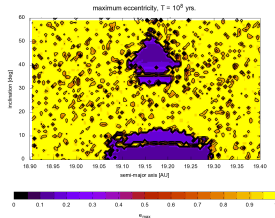


# Comparison of Methods for $T = 10^8$ yrs.

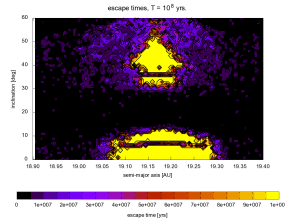
libration width



maximum eccentricity

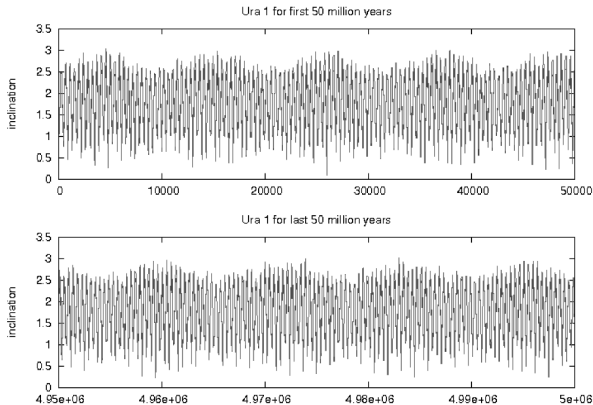


escape times



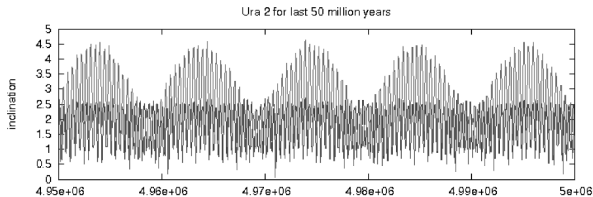
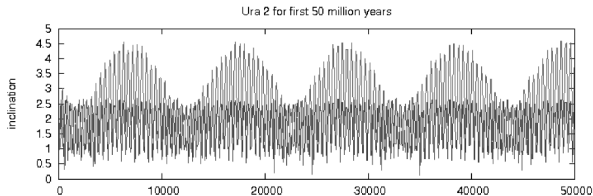


# Trojan for 5 Ga (1/3)



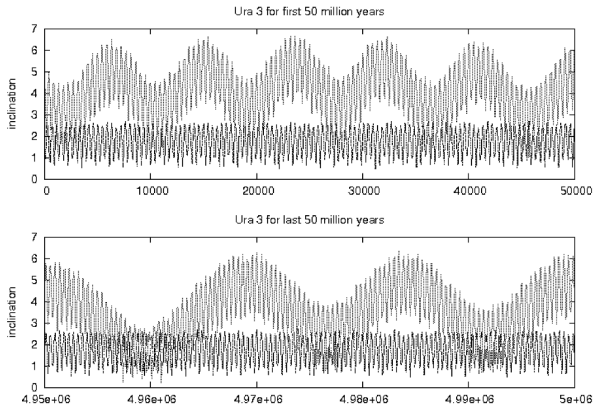


# Trojan for 5 Ga (2/3)





# Trojan for 5 Ga (3/3)





# Frequency Analysis (1/4)

About SIGSPEC

- Author: P. Reegen, A&A 467, 2007
- time series analysis by DFT
- spectral significance: accurate peak detection
- using secular variables:

$$k = e \cos(\omega + \Omega), \quad q = \sin i \cos \Omega$$





# Frequency Analysis (2/4)

Common frequencies in dynamical spectrum of Uranus

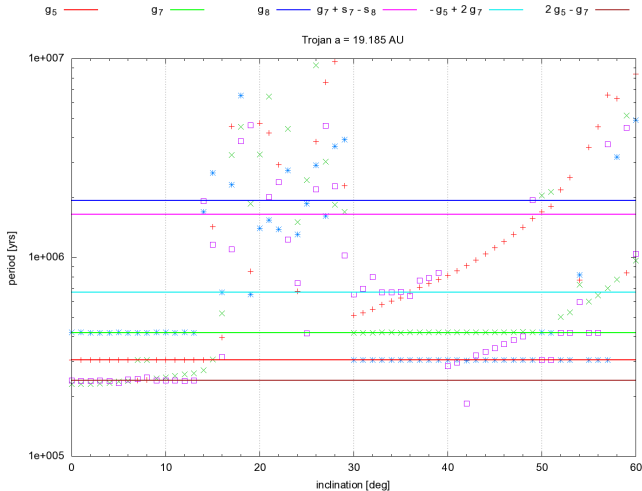
combination	periods [ $\times 10^3$ yrs]
$g_5$	305.2
$g_6$	45.7
$g_7$	419.2
$g_8$	1928.0
$g_7 + s_7 - s_8$	1650.6
$-g_5 + 2g_7$	666.8
$2g_5 - g_7$	240.1
MMR 2:1 U/N	4.236





# Frequency Analysis (3/4)

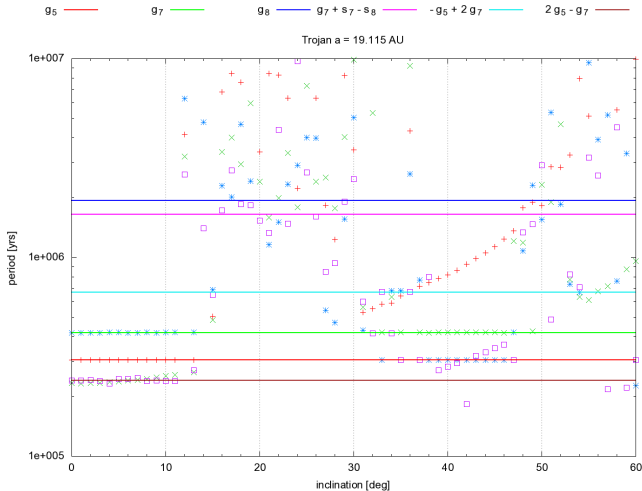
## Secular frequencies acting on Trojans





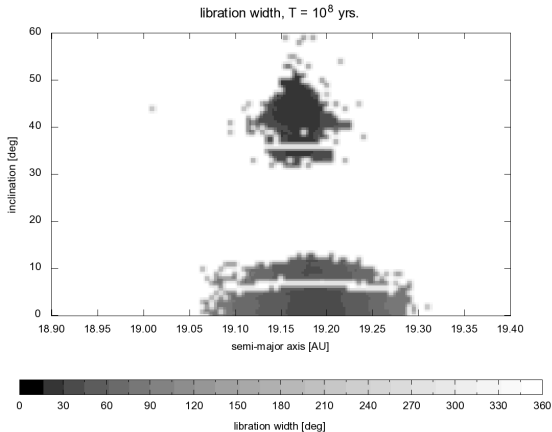
# Frequency Analysis (4/4)

## Secular frequencies acting on Trojans





# Comparison





# Conclusions

- detected **stable zones** for at least up to  $5 \times 10^8$  years
- lifetime for selected (low inclination) orbits **5 Ga**
- **secular resonances** shaping stability zone

