5th Austrian Hungarian Workshop

Frequencies of librational motions around L4

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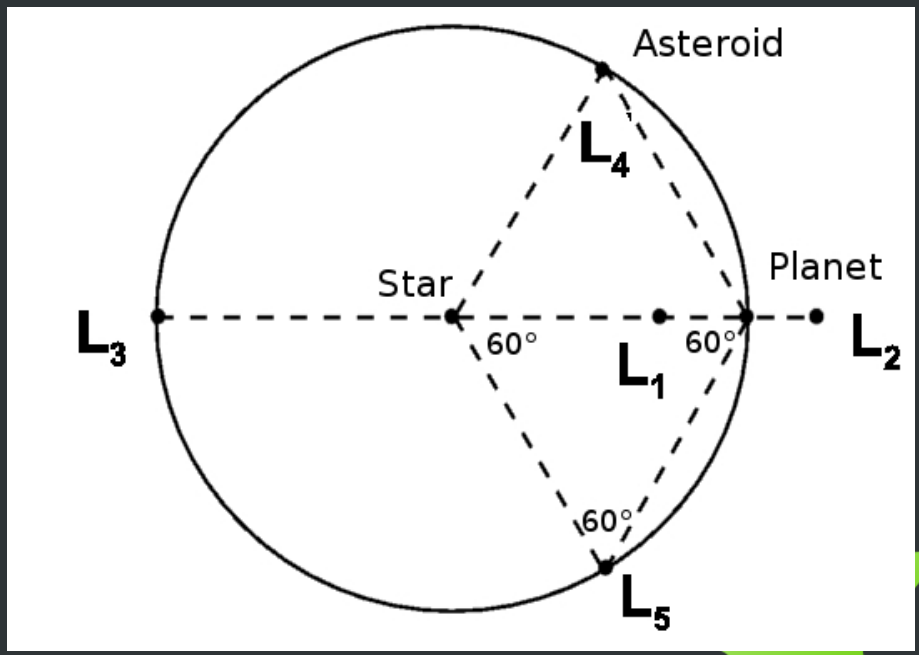
The L4 point

- The restricted three-body problem
- Star, planet, asteroid – $m_1, m_2 \gg m_3$
- Lagrange points
- Trojan asteroids on Jupiter's orbit
- Trojan bodies in the Solar system (Trojan asteroids and Trojan moons)



Frequencies of motions

- Numerical integration of the equations of motion, with different mass parameters ($\mu = \frac{m_2}{m_1 + m_2}$) and eccentricities (e) $\Rightarrow$ μ – e plane

  e:[0;1] stepsize: 0.002 **μ**:[0.0001;0.1] stepsize: 0.001

- Stable and unstable motions in the μ – e plane
- Fourier transformation – using GSL (GNU Scientific Library) routine: FFT (Fast Fourier transform)
- 4 frequencies in the stable region:

$$n_s, n_l, 1-n_s, 1-n_l$$

Frequencies of motions

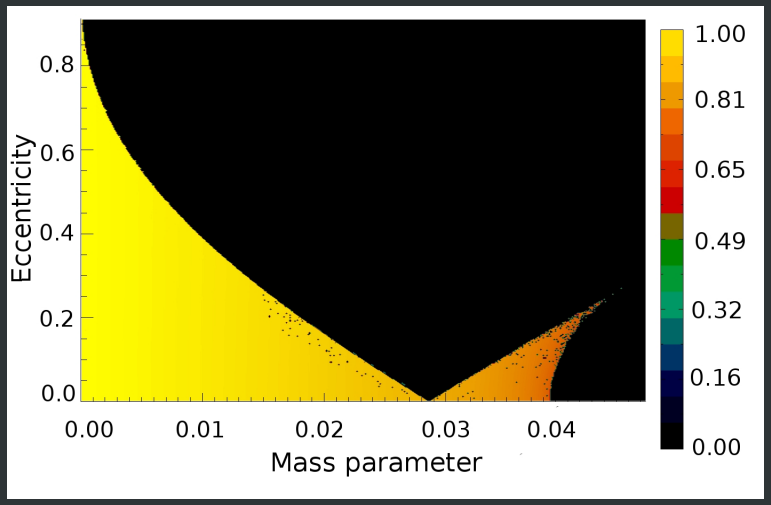
$e = 0$

μ	*analytical* - n_s	*numerical* - n_s	Δn_s
0,0001	0,999662	0,999299	0,000363
0,0002	0,999324	0,998699	0,000625
0,006	0,978763	0,97811	0,000653
0,01	0,963322	0,962517	0,000805
0,02	0,918191	0,91794	0,000251

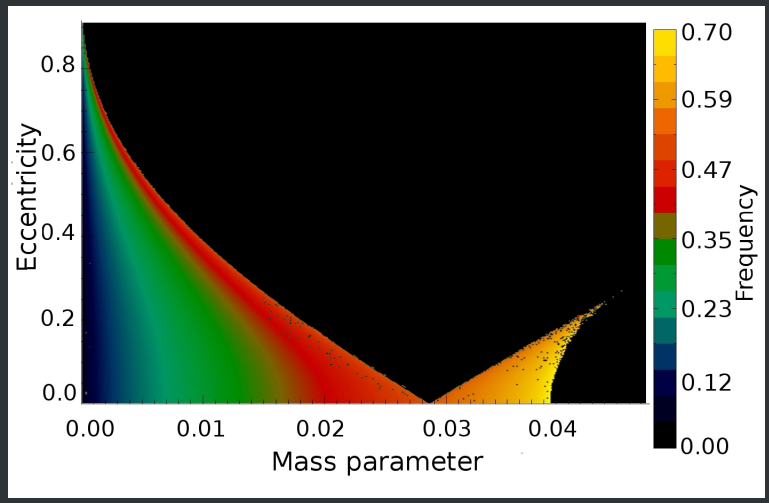
μ	*analytical* - n_l	*numerical* - n_l	Δn_l
0,0001	0,0259882	0,0257871	0,0002011
0,0002	0,0367635	0,0367816	-0,0000181
0,006	0,204995	0,204897	0,000098
0,01	0,268348	0,268066	0,000282
0,02	0,396138	0,396001	0,000137

Frequencies of motions

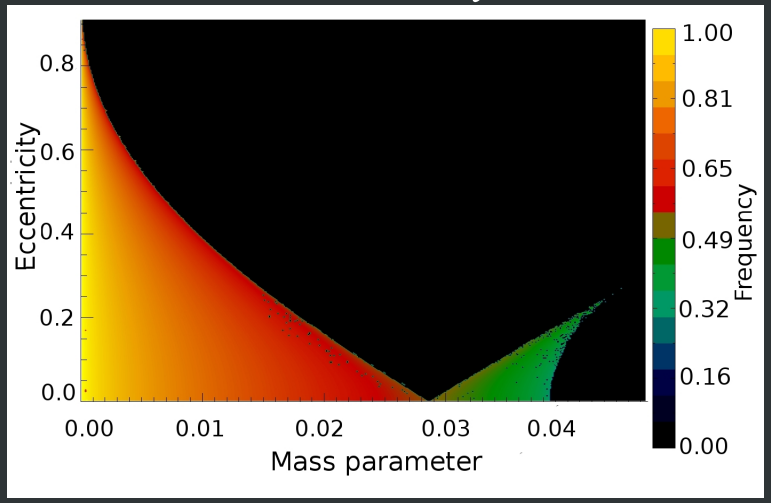
n_s



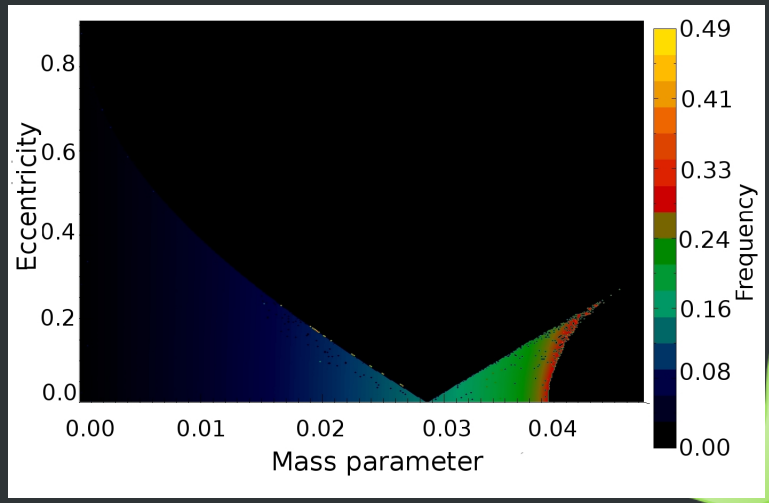
n_l



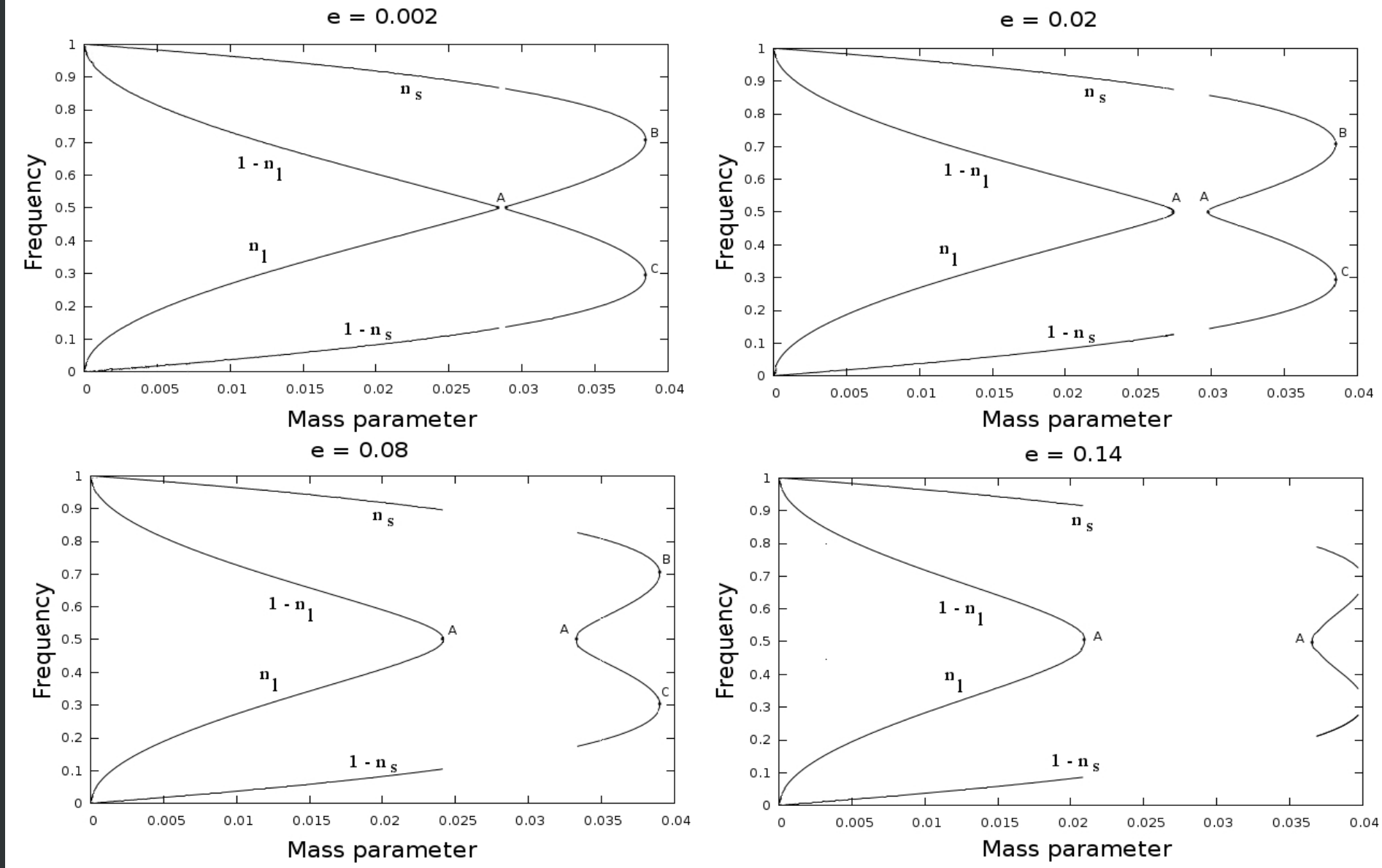
$1-n_l$



$1-n_s$



Frequencies of motions

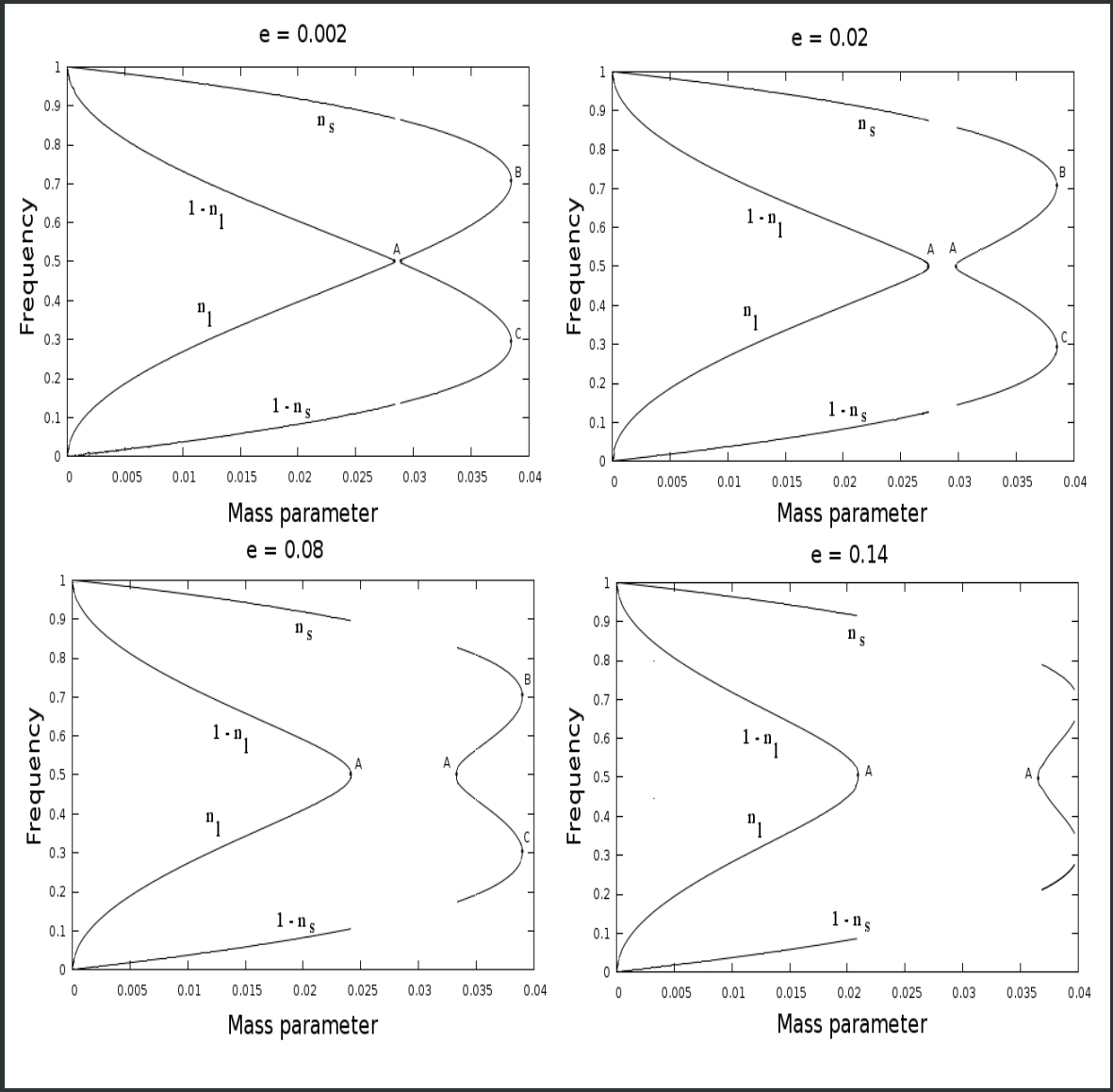


Resonances

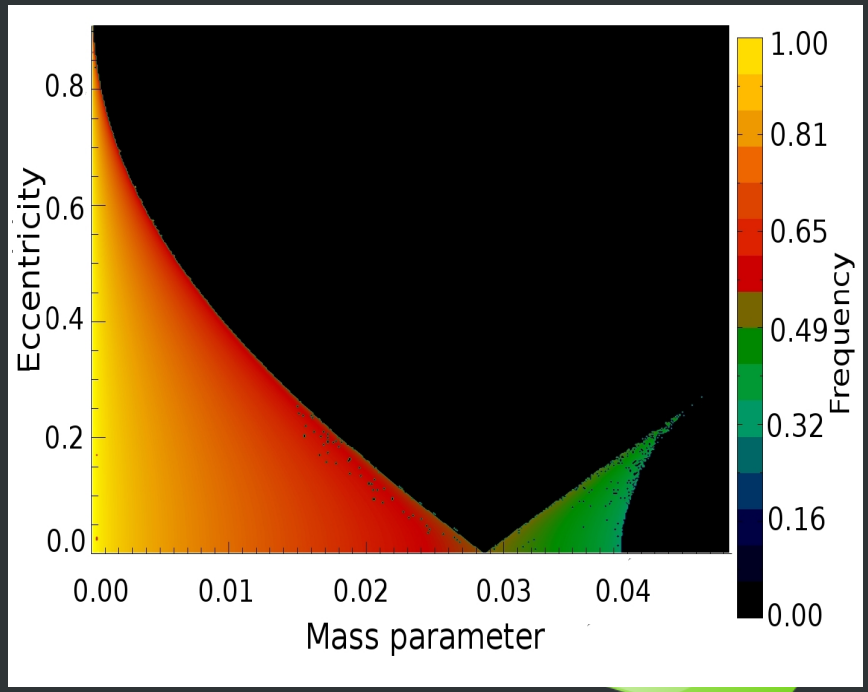
- Resonances: when the quotients of frequencies are rational numbers.
- Data near the resonances with ±0.001
- 6 types:

A	$(1 - n_l) : n_l$
B	$n_s : n_l$
C	$(1 - n_l) : (1 - n_s)$
D	$n_s : (1 - n_l)$
E	$n_s : (1 - n_s)$
F	$n_l : (1 - n_s)$

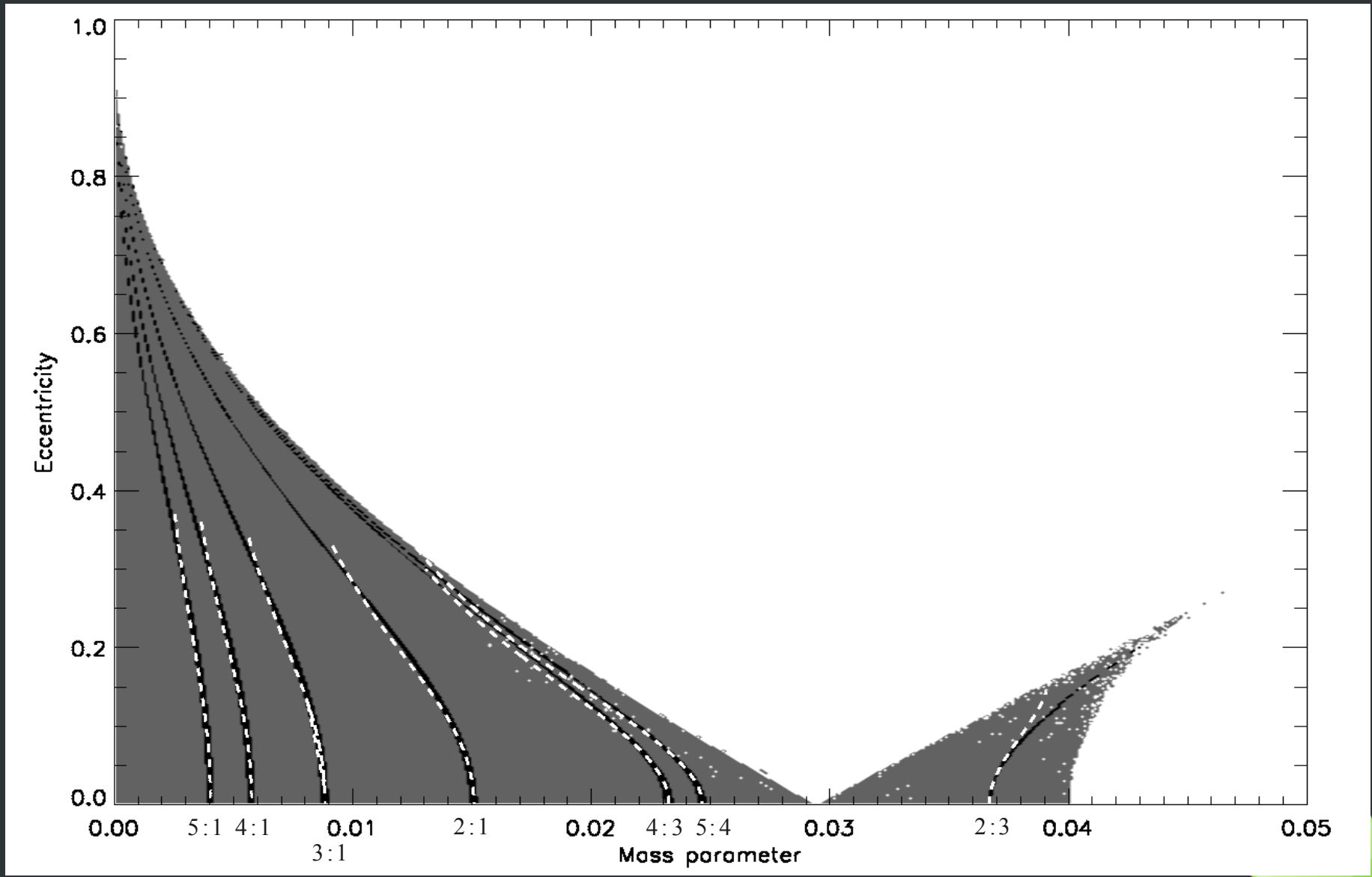
Resonances



A	$(1 - n_l) : n_l$
B	$n_s : n_l$
C	$(1 - n_l) : (1 - n_s)$
D	$n_s : (1 - n_l)$
E	$n_s : (1 - n_s)$
F	$n_l : (1 - n_s)$



Resonances


1.0
0.8
0.6
0.4
0.2
0.0
Eccentricity
0.00
5:1
4:1
3:1
0.01
2:1
0.02
4:3
5:4
0.03
2:3
0.04
0.05
Mass parameter

Future work

- Determine the frequencies in the unstable region with other method

  See if it fills the lack of data

- What kind of affect resonances have

  If they have intersections