

AUSTRIAN-HUNGARIAN

AUSTRIAN
WORKSHOP 2002

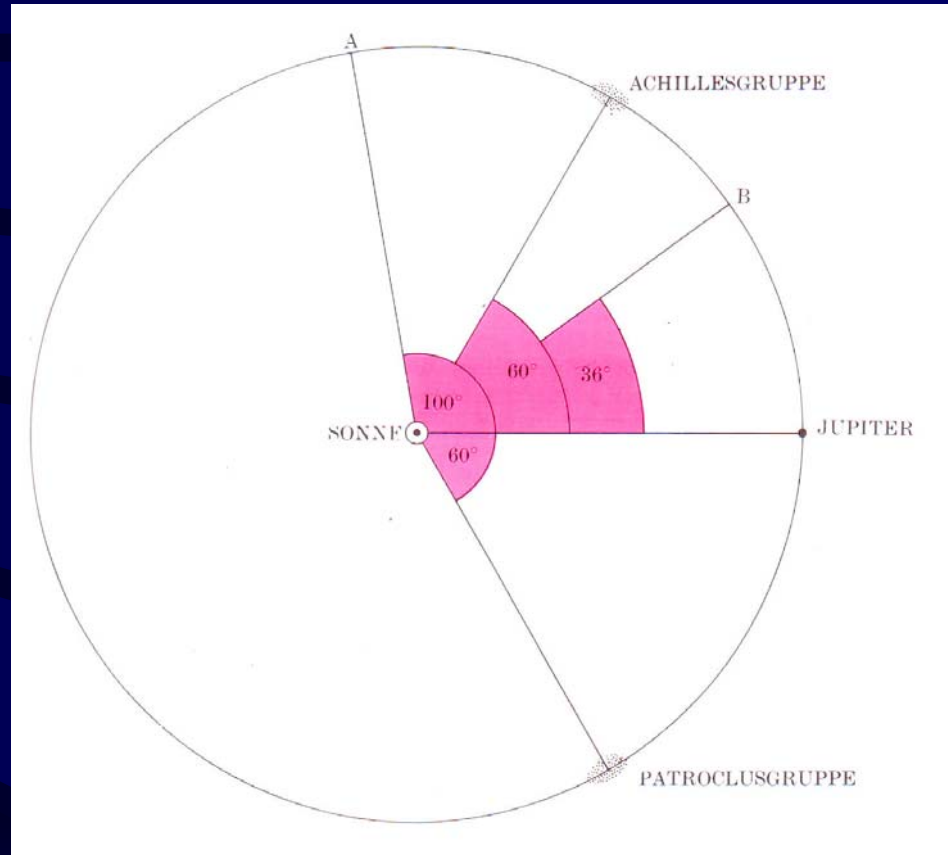
Stability of Trojans with high inclined
orbits

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- Introduction
- Numerical setup
- Fast orbit classification
- Classification of stability
 - .) Running window Averaging
 - .) Results
- Histogram
- Discussion

Introduction

- 1) Nowadays (April 2002)
696 Asteroids at
equilibrium point L_4
- 2) 519 Asteroids at the
Lagrangian point L_5
are known



Numerical setup

- .) 23 Asteroids L_4 , 18 Asteroids L_5 for 50 Myrs
- .) Group of the fiktive Asteroids have a small deviation of the semi major axis ($\Delta a = \pm 0.01 \text{ AU}$), and excentricity ($\Delta e = \pm 0.01$).
- .) Calculations by **ORBFIT** (Milani)
- .) Dynamical system: was the Outer Solar System Model (OSS:Sun+outer planets)

The output data of the calculations are:

- .) the semi-major-axis (a), and
- .) the vertical and horizontal component of the eccentricity and inclination (h, k, p, q).

From this equations:

$$h = e \sin \varpi, \quad k = e \cos \varpi,$$
$$p = I \sin \Omega, \quad q = I \cos \Omega,$$

...follows the requested orbital elements i and e :

$$i = \sqrt{p^2 + q^2}, \quad e = \sqrt{h^2 + k^2}.$$

Fast orbit classification

The libration width σ is defined as the difference of the main longitude of the asteroid and Jupiter ($\lambda - \lambda_J$).

$$\lambda = \varpi + M, \quad \lambda_J = \varpi_J + M_J$$

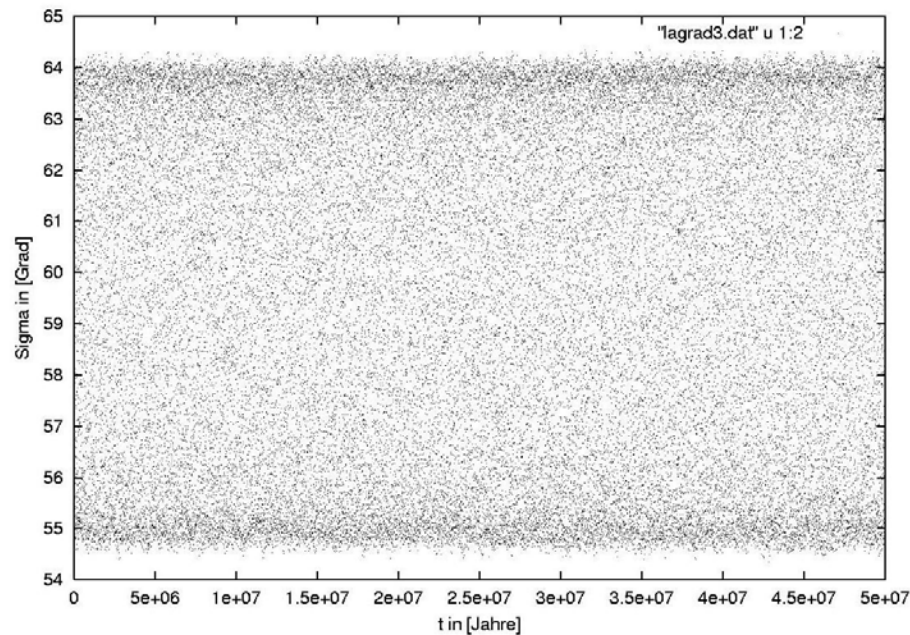
ϖ ...longitude of the asteroid

ϖ_J ...longitude of Jupiter

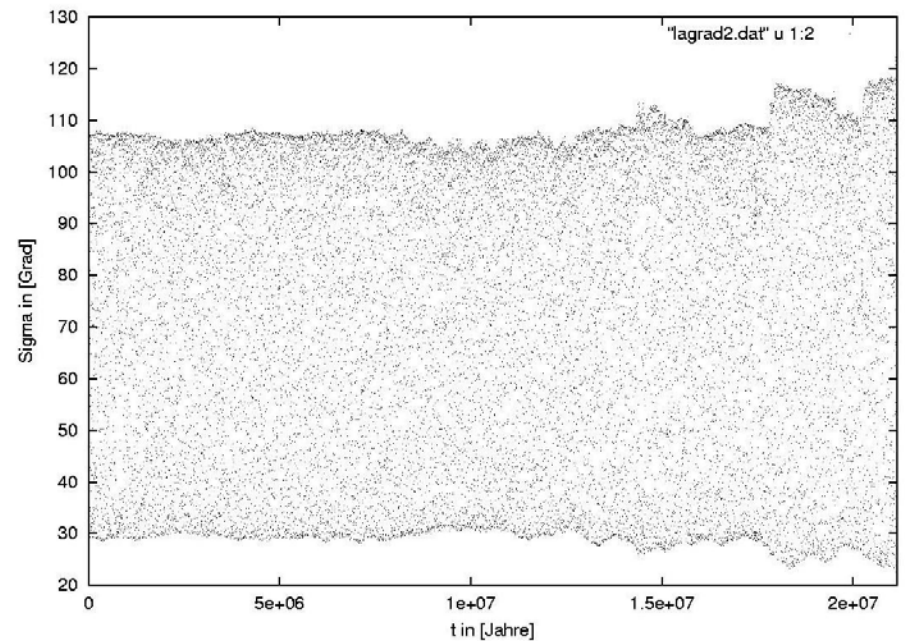
M ...mean anomaly of the asteroid

M_J ...mean anomaly of Jupiter

For this classification I only took into account the data of the original asteroids.



pic1) Libration width of Orsilocus



pic2) Libration width of Asteroid 15539

Classification of the stability

.) Running-Window-Averaging

This method consist of the following steps:

- .) Divide the whole integration time into equal parts
- .) Shifting the window along the time axis
(with certain overlapping regions) and calculates averaged and rms values of e and i for each shift
- .) Classification:

Classification through the expression:

$$\Delta \langle e \rangle - \delta(e) = (\langle e \rangle_{\max} - \langle e \rangle_{\min} - \langle rms(e) \rangle),$$

$$\Delta \langle i \rangle - \delta(i) = (\langle i \rangle_{\max} - \langle i \rangle_{\min} - \langle rms(i) \rangle).$$

- .) **Stable** orbit: if both expressions are negative
- .) Otherwise the orbit is said to be **unstable**

Results

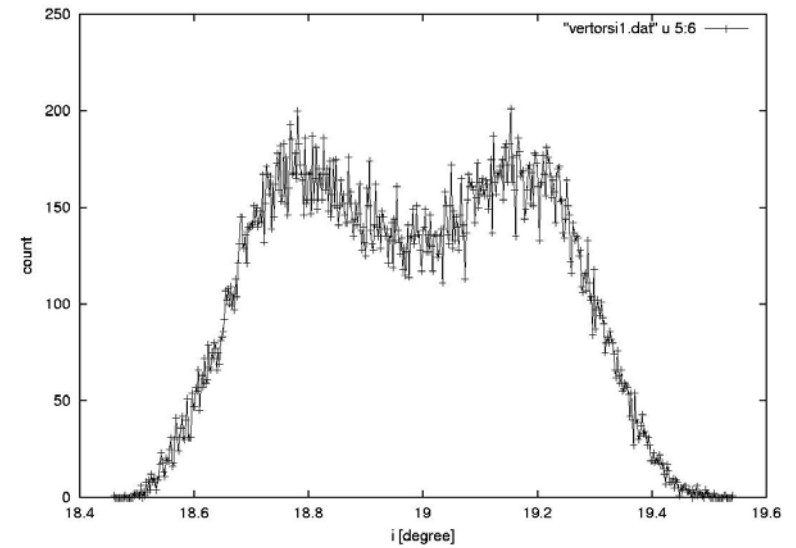
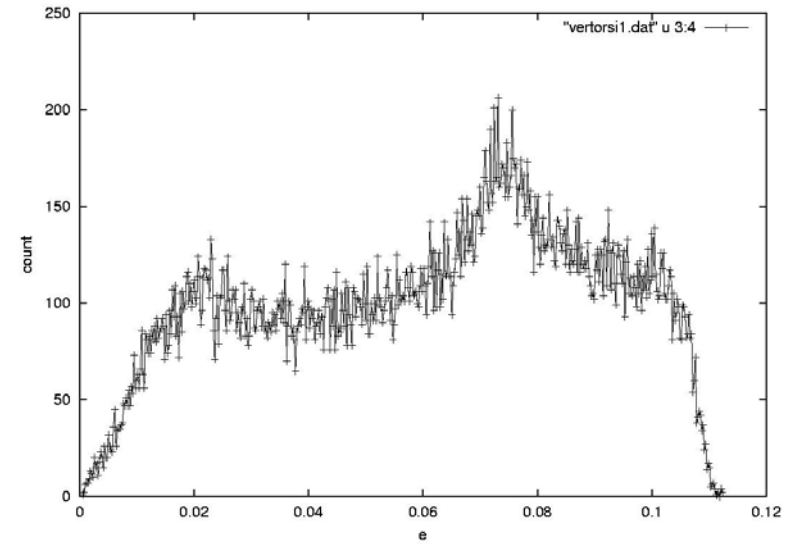
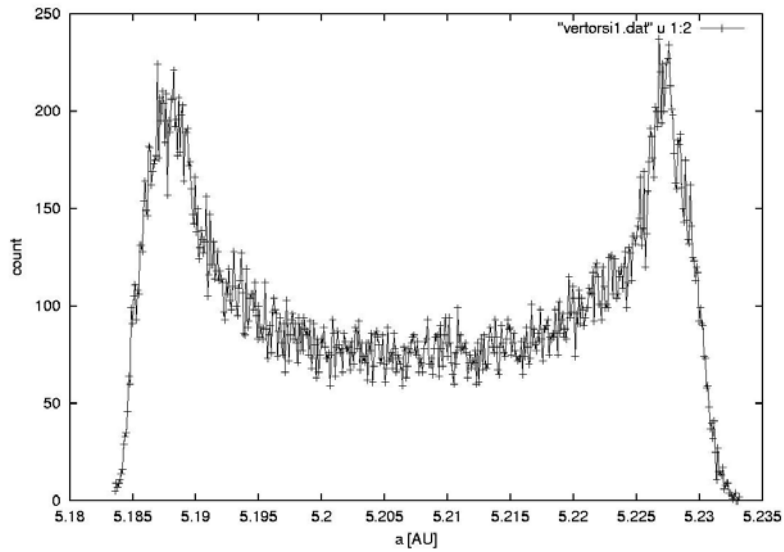
Lagrangian point L4	number	stable	unstable	escaping orbits
number of asteroids	23	15	7	1
number of neighbours	92	66	24	2
all objects	115	81	31	3

Lagrangian point L5	number	stable	unstable	escaping orbits
number of asteroids	18	9	9	0
number of neighbours	72	40	31	1
all objects	90	49	40	1

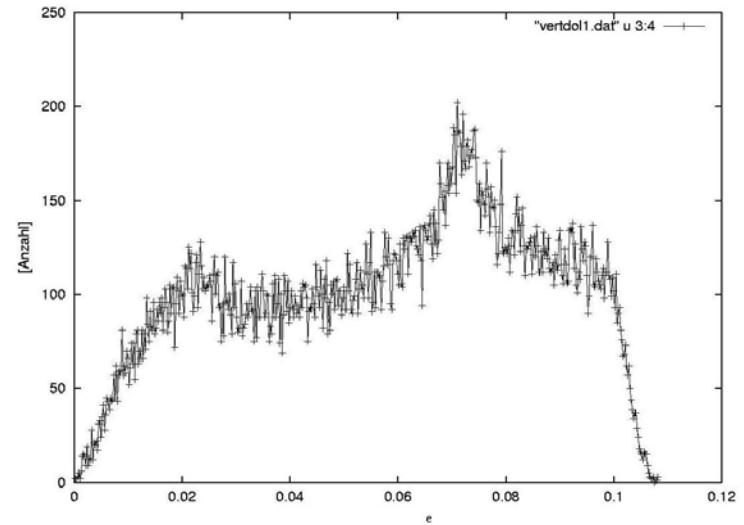
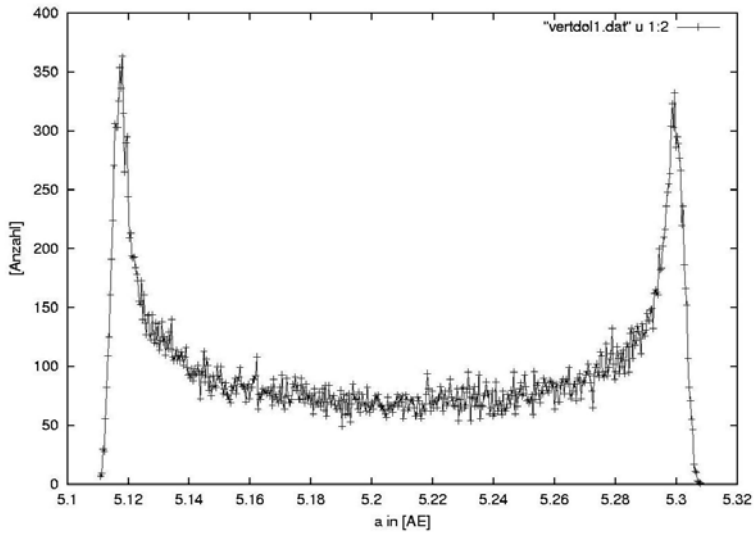
Histogram

Due to the perturbation of the other planets, the orbital elements a, e, i are not constant. Therefore were a **histogram** performed, in order to get informations about the variations of the orbital elements.

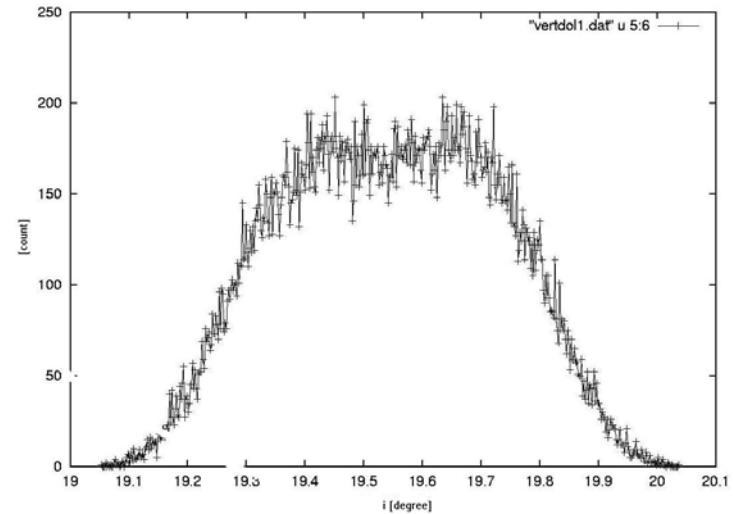
To get the histogram it is necessary to divide the difference of the maxima and minima values of the orbital elements into classes. Then determine the number of the values of each class.

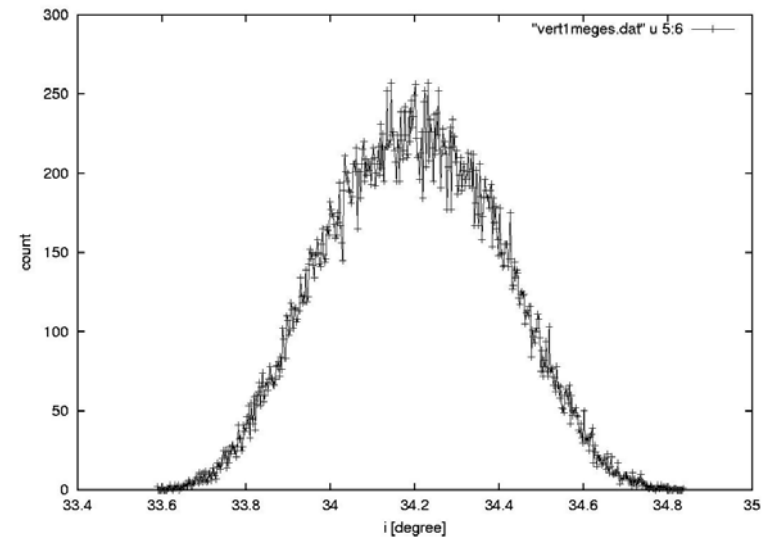
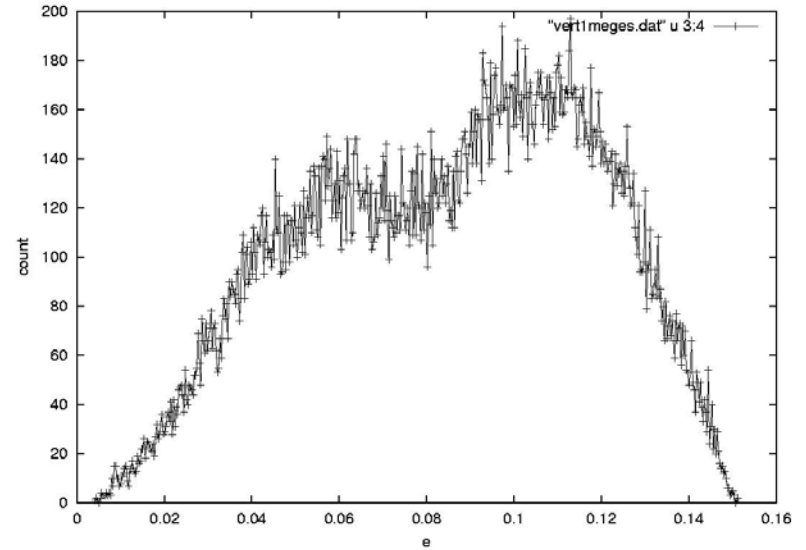
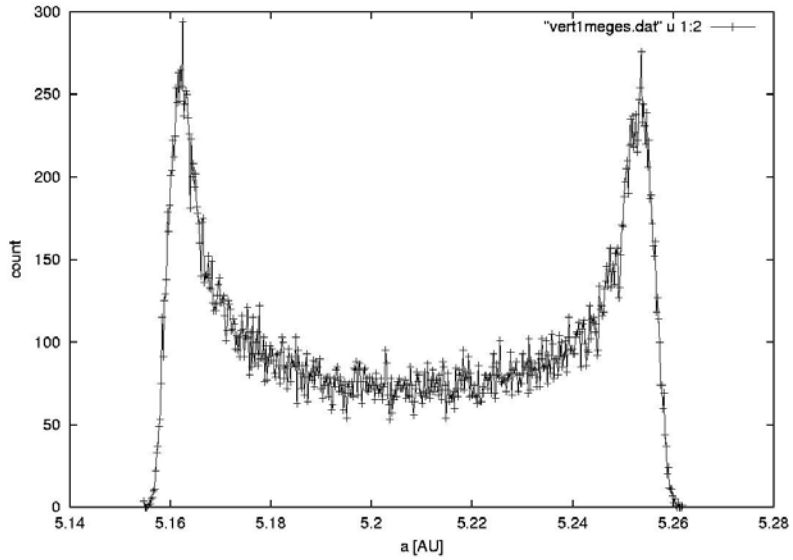


Histogram of the **stable**
asteroid Orsilocus (L_4)

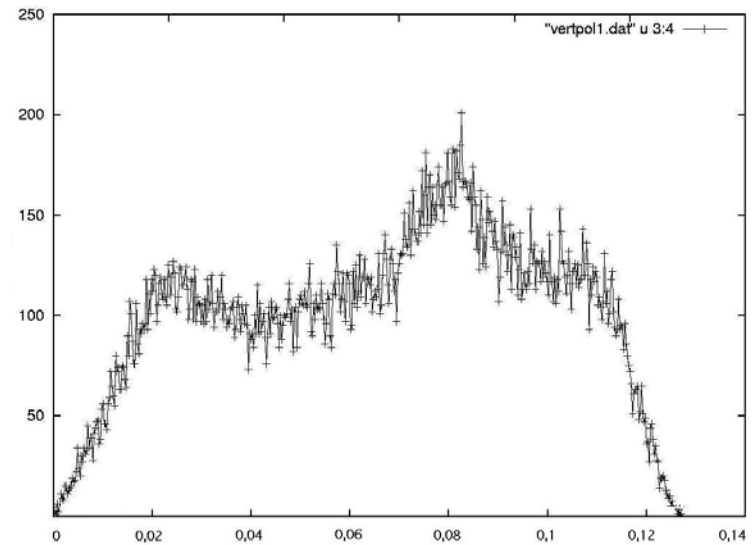
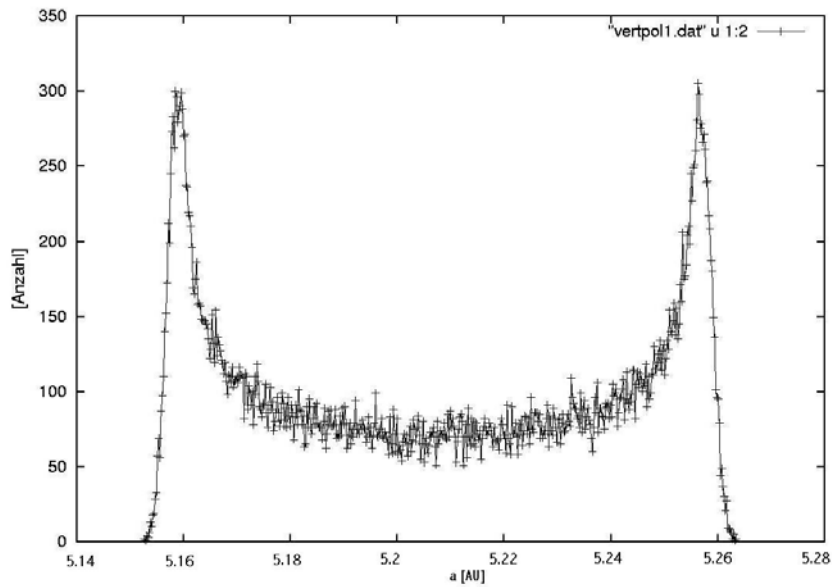


Histogram of the **stable**
 asteroid Dolon (L₅)

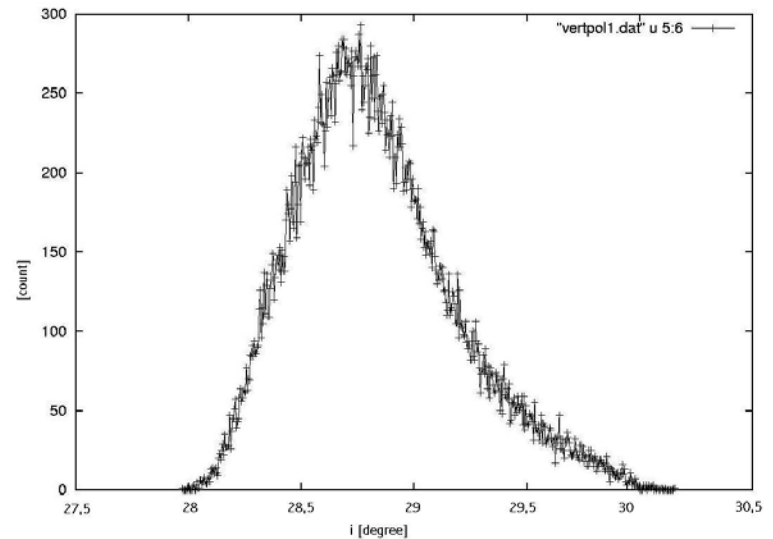


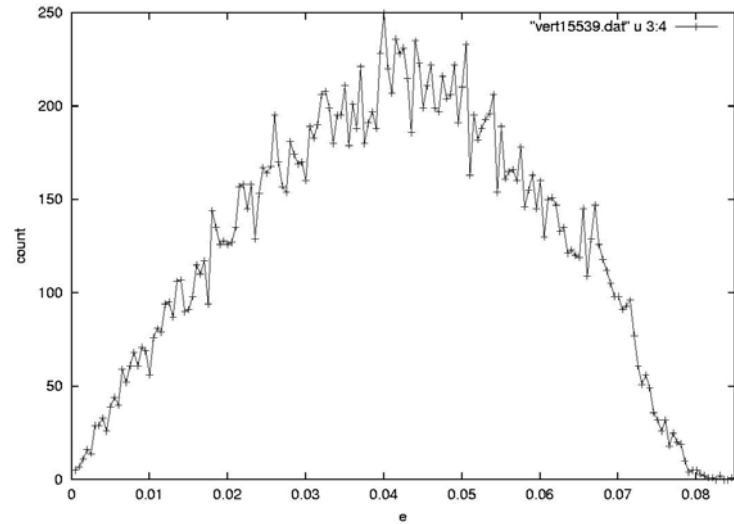
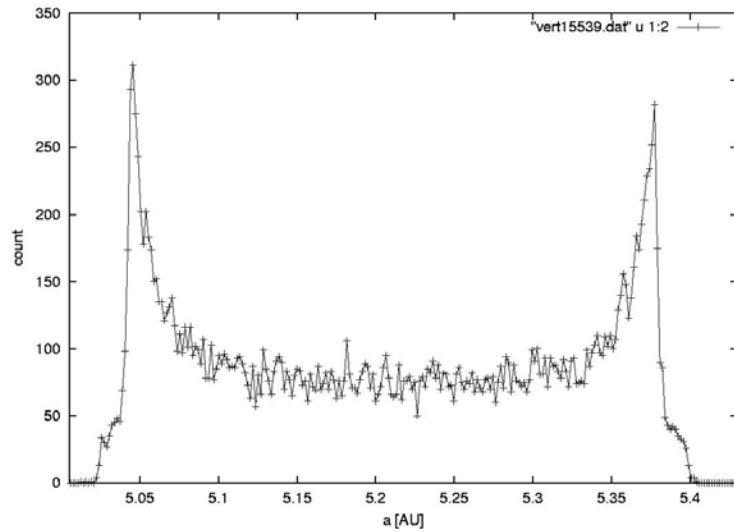


Histogram of the **unstable** asteroid Meges (L₄)

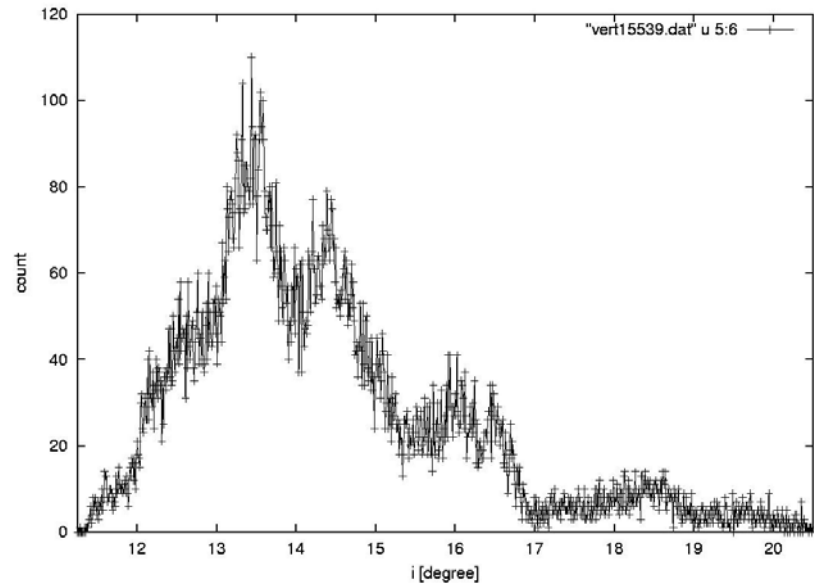


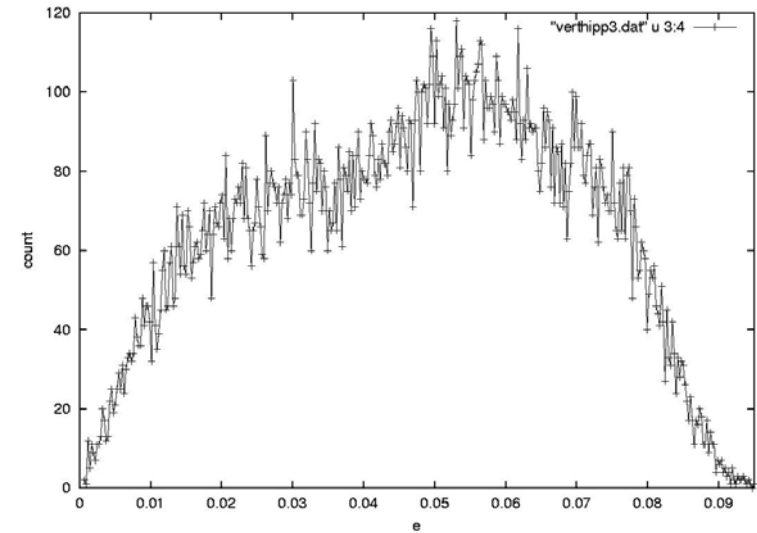
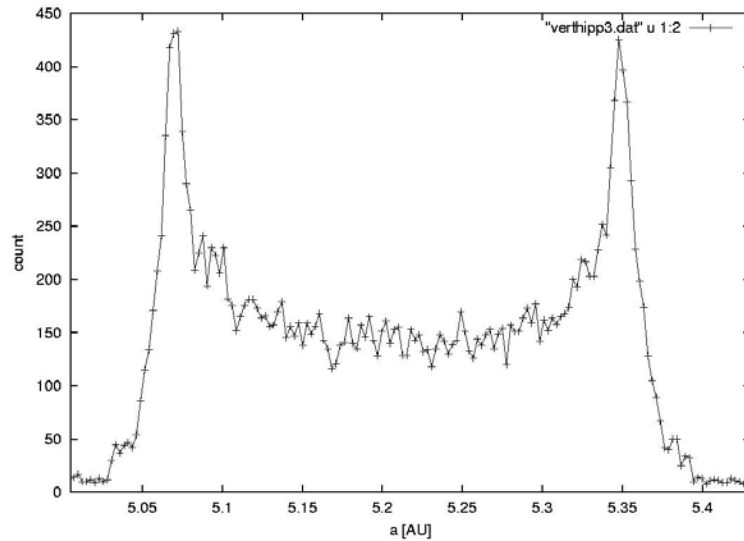
Histogram of the **unstable** asteroid Polites (L_5)



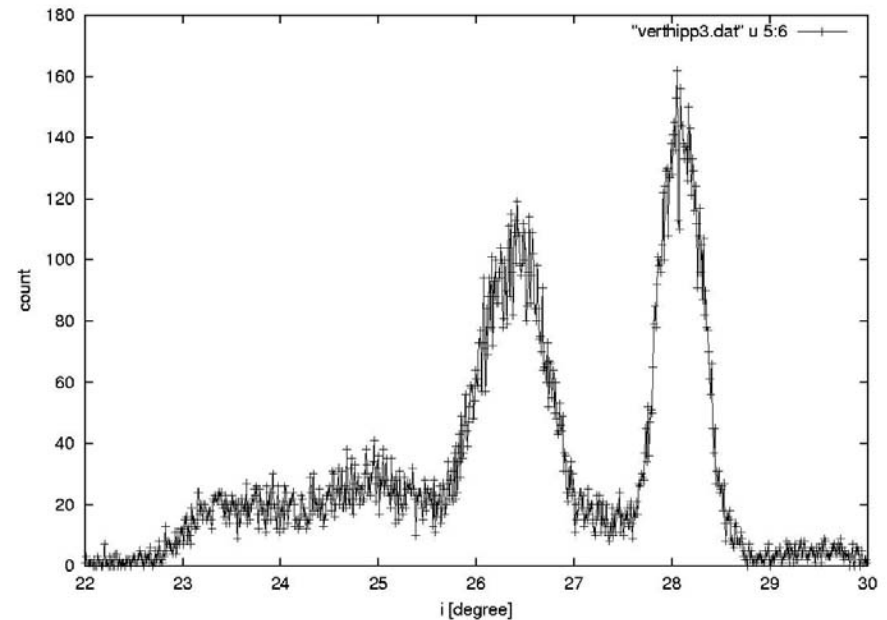


Histogram of the **escaping** asteroid 15539 (L_4)



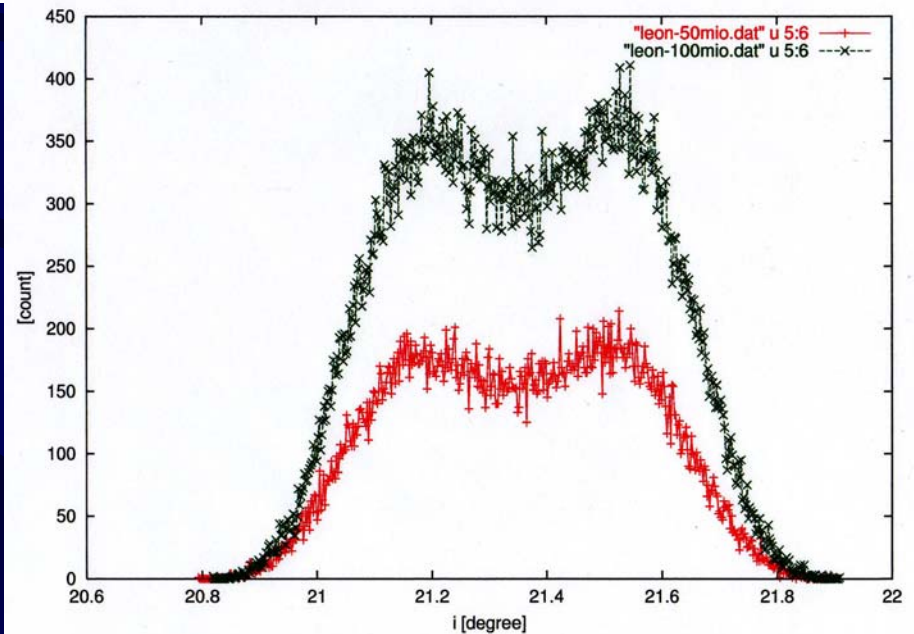
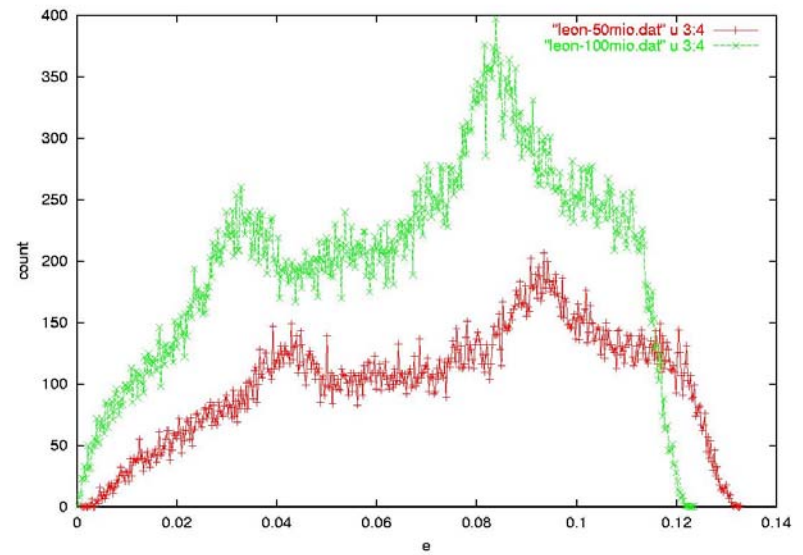
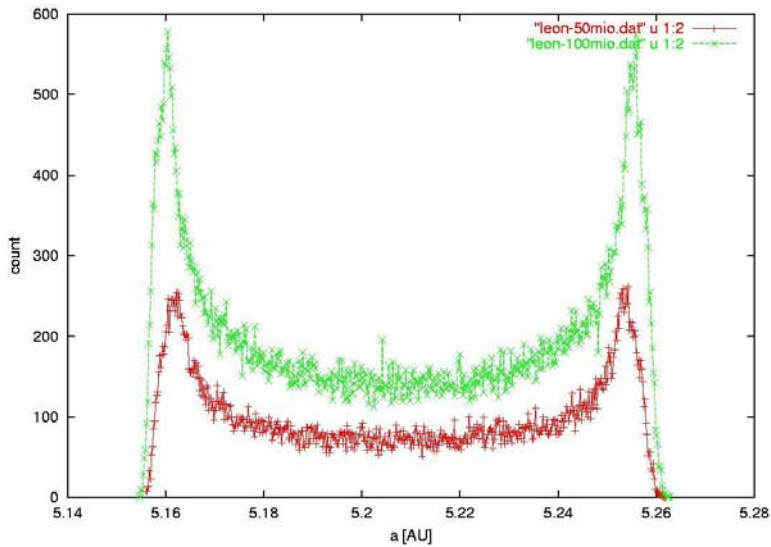


Histogram of the **escaping** asteroid Hippasos (L_5)

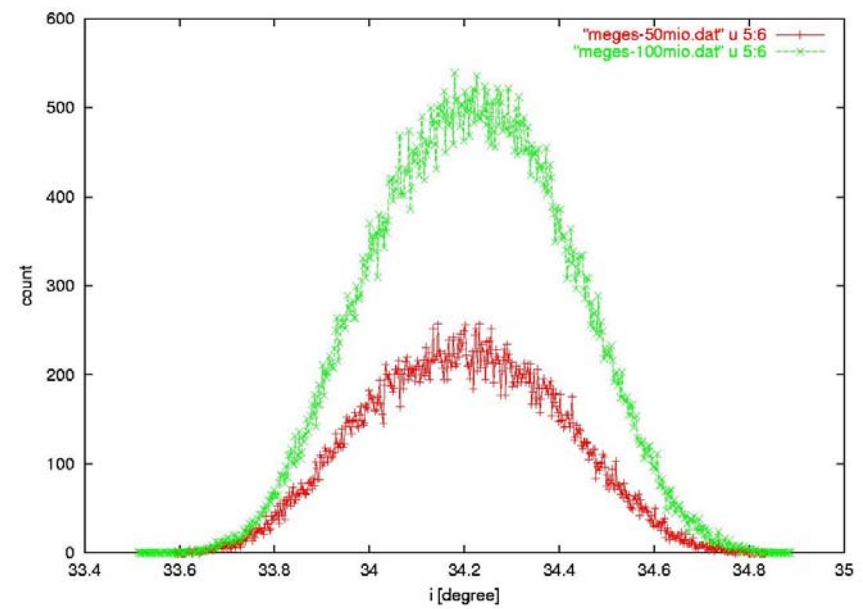
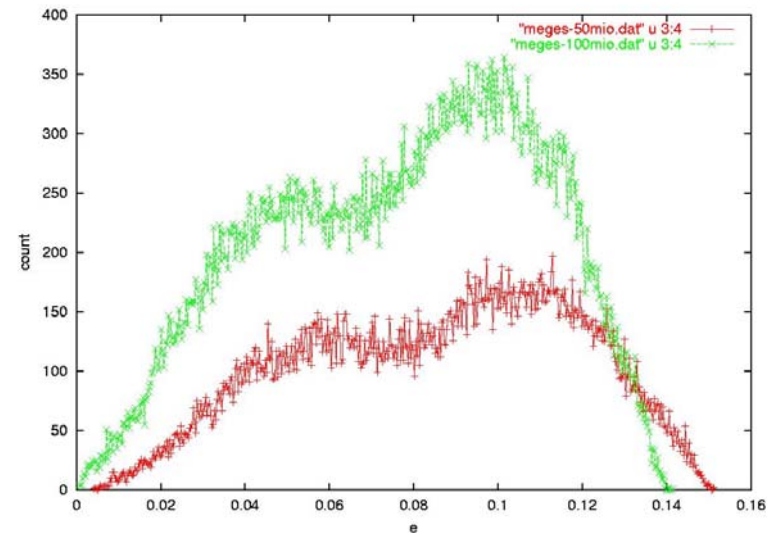
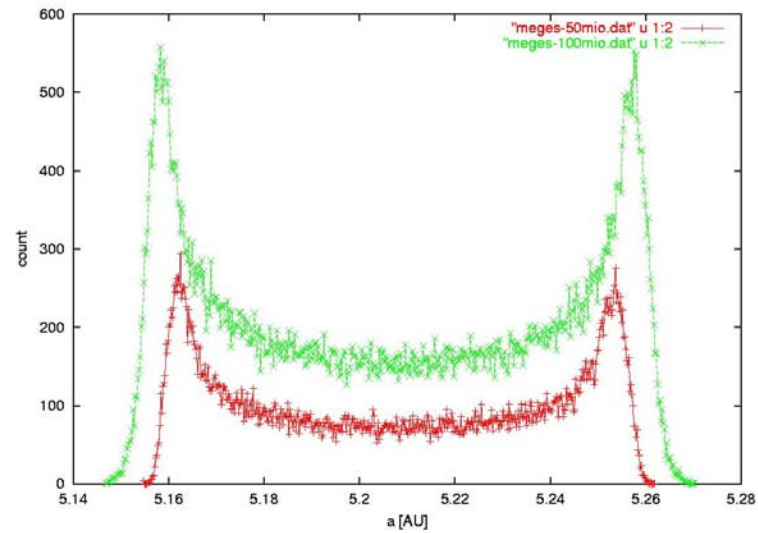


Discussion

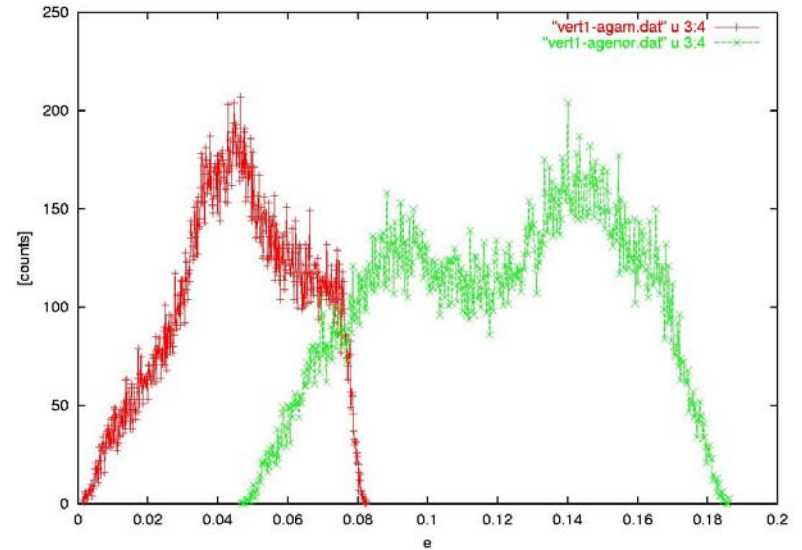
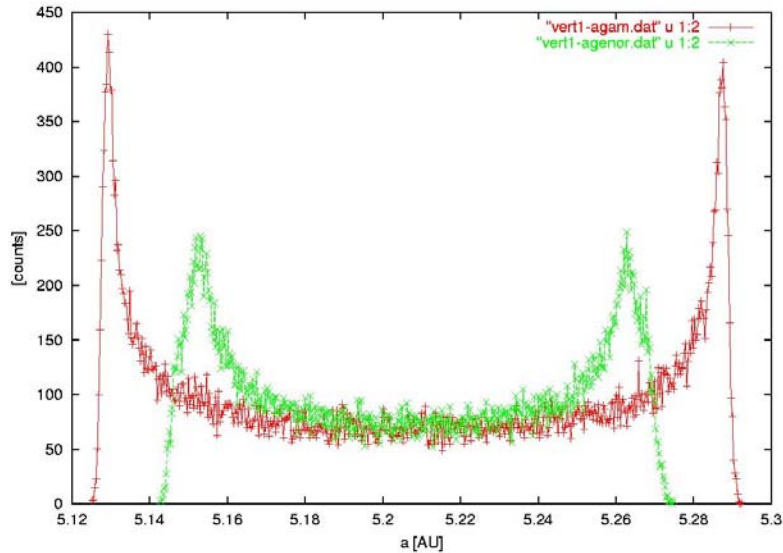
- Comparison of Asteroids with different calculation-time
- Comparison of Asteroids of different Lagrangian points with the same initial condition



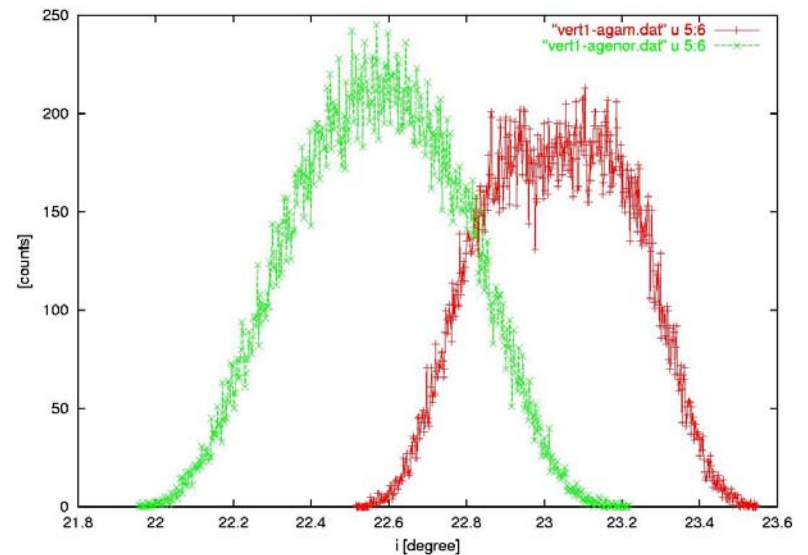
Asteroid Leonteus (**stable**)
 calculated for 50 and
 100Myrs



Asteroid Meges (**unstable**)
 calculated for 50 and
 100 Myrs



Comparison of **Agamemnon** (L_4)
 initial condition $a=5.242$ AU,
 $e=0.067$ and $i=21.799^\circ$
 and **Agenor** (L_5)
 initial condition: $a=5.258$ AU,
 $e=0.90$ and 21.848°



FINISH