On the stability of the habitable zones of exoplanetary systems

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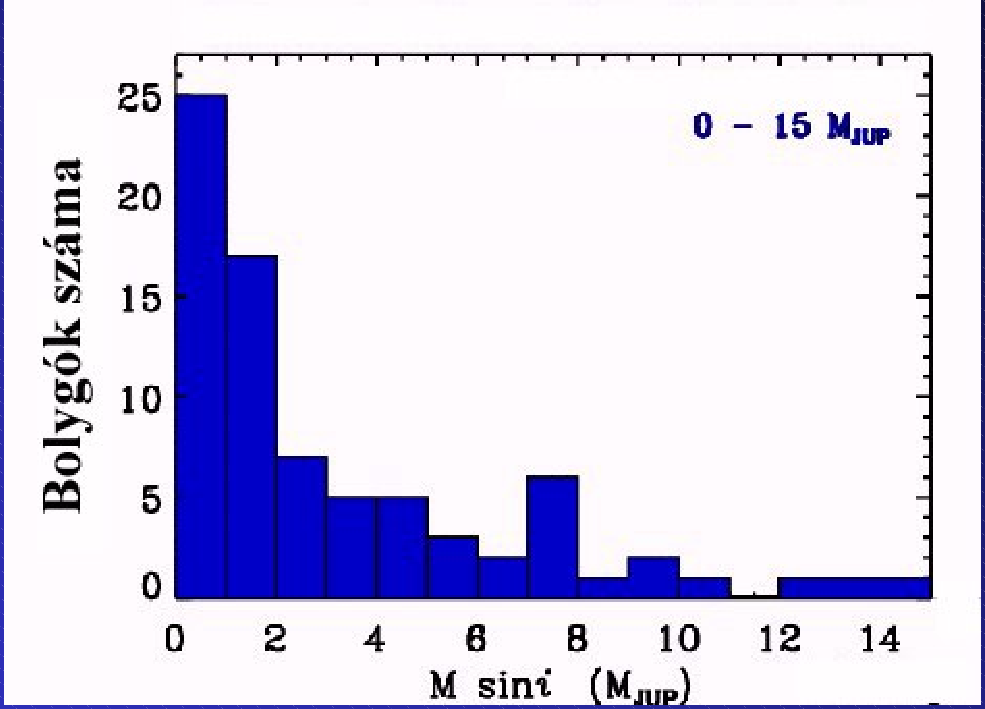
Exoplanetary systems - overview

- Since the discovery of the first extrasolar planet, up to now more than 70 exoplanetary systems are known
- Are there physical conditions for Earth-like life in such systems?
  - Investigations from the point of view of celestial mechanics: dynamical stability, regular and chaotic motions
  - Our results agree well with the previous ones, but give more global picture of the investigated systems
- Our Solar System can serve as a laboratory of the exoplanetary systems

Distribution of known systems

- Dominance of the planets close to the central stars
- Selection effect: by using the recent observing techn-iques, the massive and nearby planets can be discovered easier



Question of the habitable zone

- One of the motivations to study the exoplanetary systems
- Conditions for Earth-like life:
  - the presence of liquid water on the surface of the planet
  - the temperature can be provided by the atmosphere and greenhouse effect
- Definition of the habitable zone: Kasting et al, 1993.

Our applied method:

- Main problem:
  - stability: which inital conditions result in regular (or chaotic) orbits?
  - Orbits are obtained by numerical integration of the equations of motion (ordinary differential equations)
  - Quantities to describe the chaotic behaviour, these can be calculated by numerical methods
- Aim: map the phase space for a large set of initial conditions - where are the regular and chaotic regions?

Lyapunov-indicators

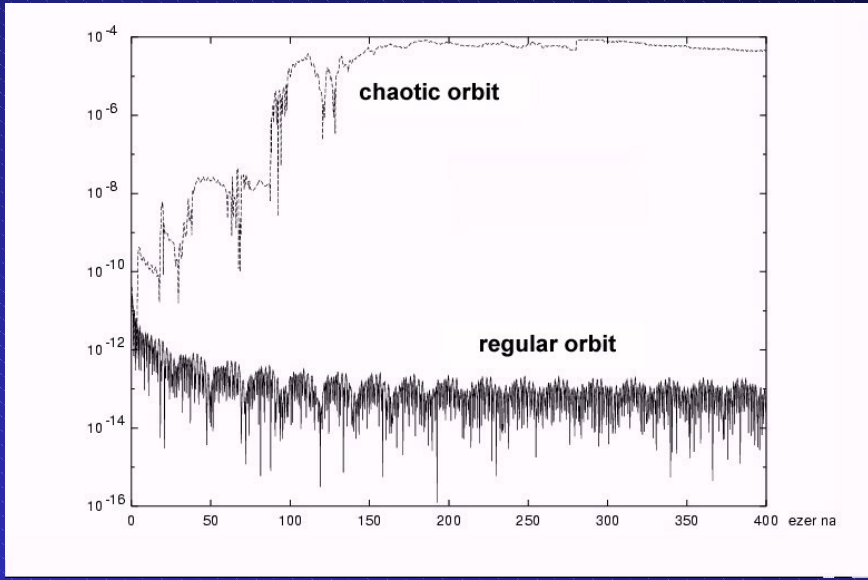
- Main method: solve the original equations and their linearized equations together to calculate the maximum *Lyapunov characteristic exponent*
- The maximum Lyapunov characteristic exponent (LCE)
  - in regular case is zero
  - in chaotic case is a positive number
- Disadvantage of the method: in the case of the weak chaos LCE is a very small number, and it can be obtained after very long time numerical integration

Relative Lyapunov-indicators

- Idea: take two very close trajectories and calculate the finite time approximation of the LCE
- The *relative Lyapunov-indicator* (RLI) is defined as the difference of these two indicators
- The RLI provides a very effective tool for detection chaotic behaviour in a very short integration time

Relative Lyapunov-indicators

- The behaviour of the RLI can be studied as the function of time during numerical integration:
  - In the case of regular orbits the RLI oscillates around a small value falling into the magnitude of the initial separation
  - In the case of chaotic orbits the curve grows rapidly

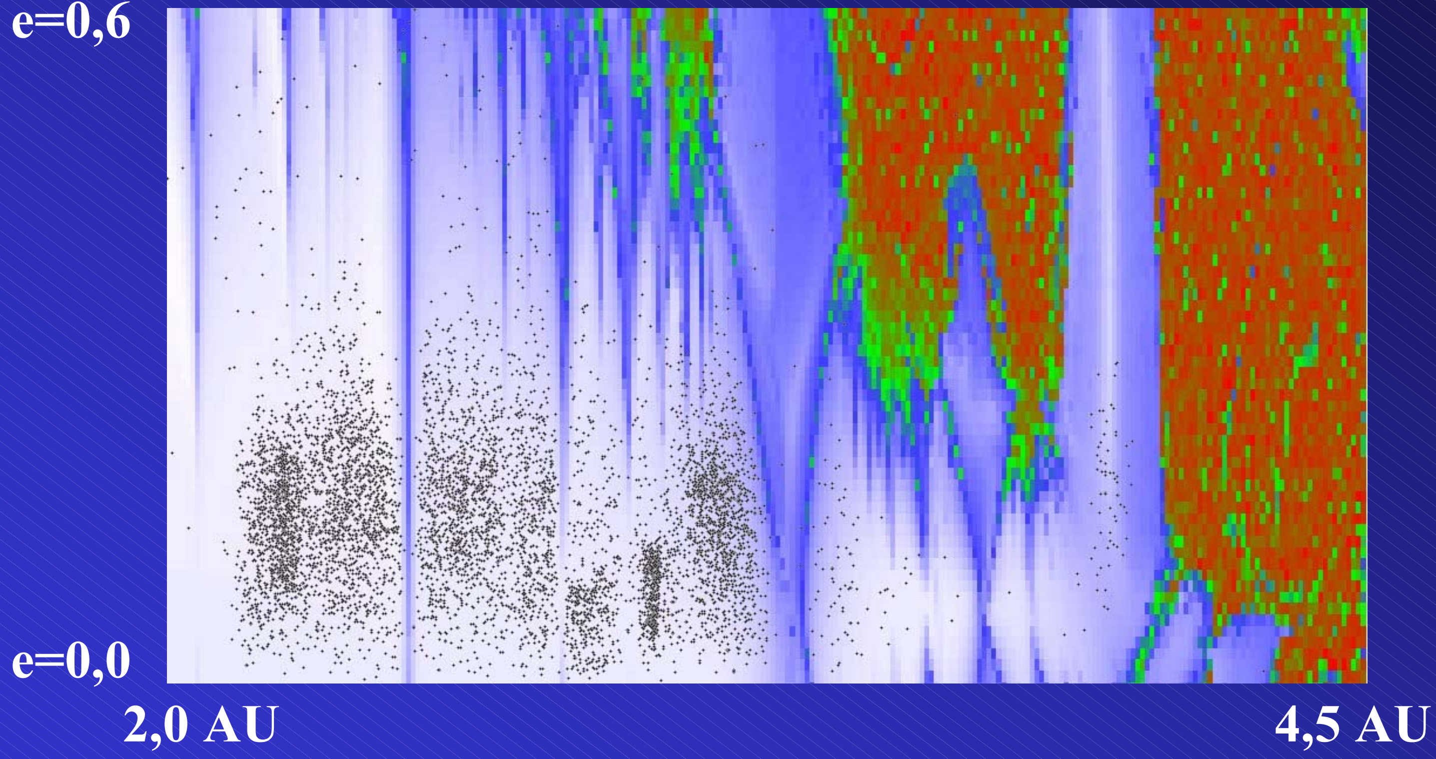


The applied numerical program

- Calculating the RLI in the N-body problem. In the case of RTBP: the motion of the perturbing planet is calculated exactly
- The LI and RLI can be calculated for planets with mass and for test particles
- Calculations can be performed in two and three dimensions
- Any parameter (mass or orbital element) can be changed automatically - the phase space is mapped with different initial conditions:
  - a, e: general stability
  - a, ϖ: studying L4 and L5 Lagrange-points
  - a_0, e_0: changing the elements of the perturbating planet(s)

Application to the Solar System

- Study the distribution of minor planets in the main asteroid belt in the Sun-Jupiter-Saturn-asteroid restricted four-body problem: the chaotic regions are dark blue, green and red, the dots represent the known minor planets (up to 7000) occupying the regular regions (white)



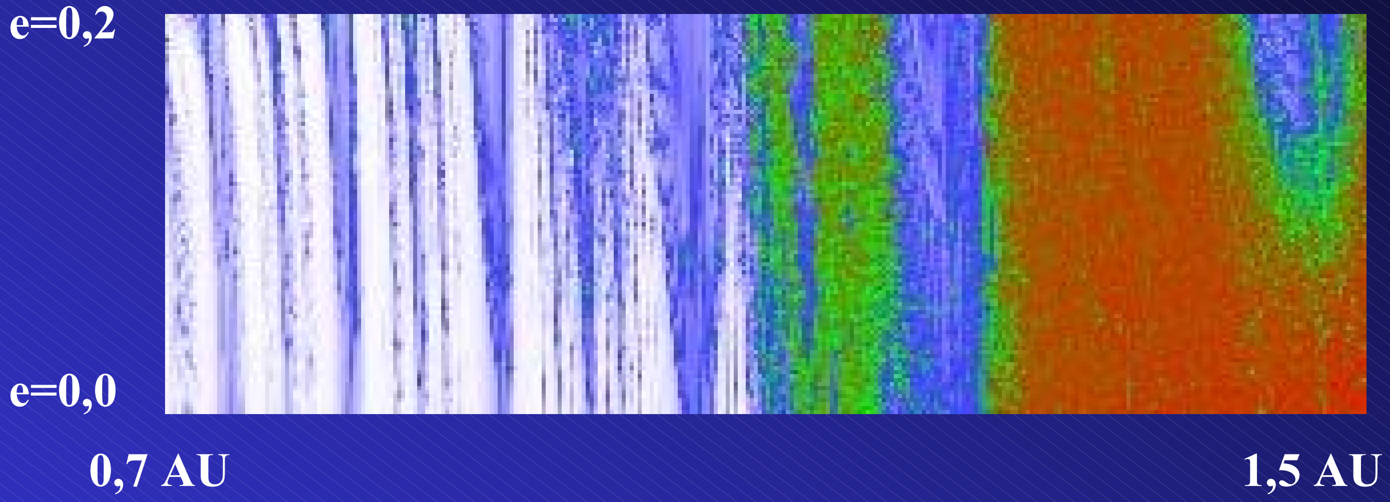
Investigation of exoplanetary systems:

- The studied systems:
  - 14 Herculis
  - 47 Ursae Maioris
  - ε Eridani
  - 70 Virginis
  - HD 80606
  - ρ Coronae Borealis
  - υ Andromedae
- Types of systems: inner, outer planets and multiple-systems

Examining criteria:

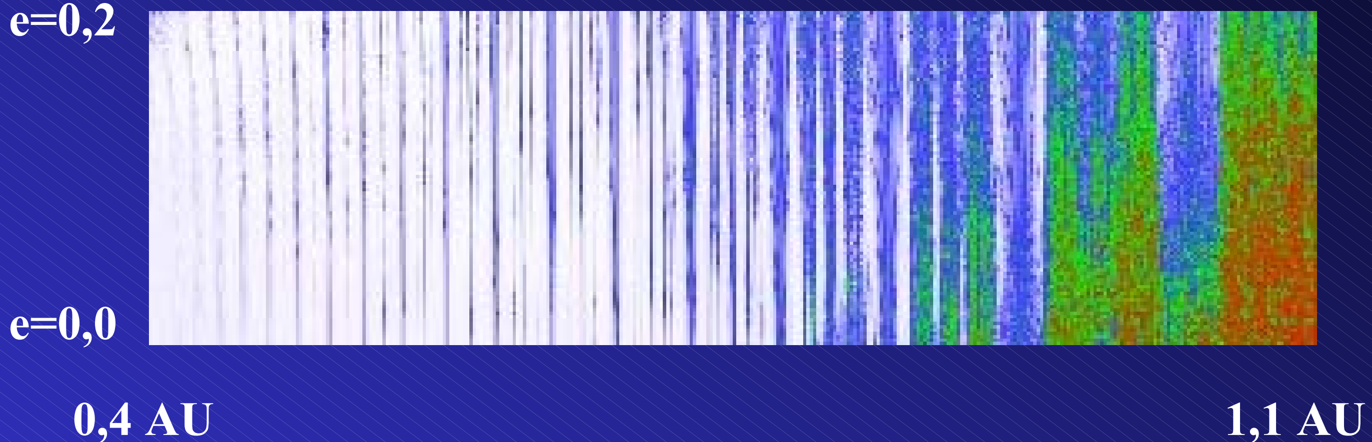
- Mapping the phase space in the region
  - in semimajor-axis: (0,6 ... 1,5 AU)
  - in eccentricity: 0 ... 0,2
- Perturbing planet(s) and its mass(es):
  - Just a lower limit -- $M \sin(i)$ -- is known, thus some other examinations shoud be made with different mass values
  - used mass values: minimal and possible mass

14 Herculis:



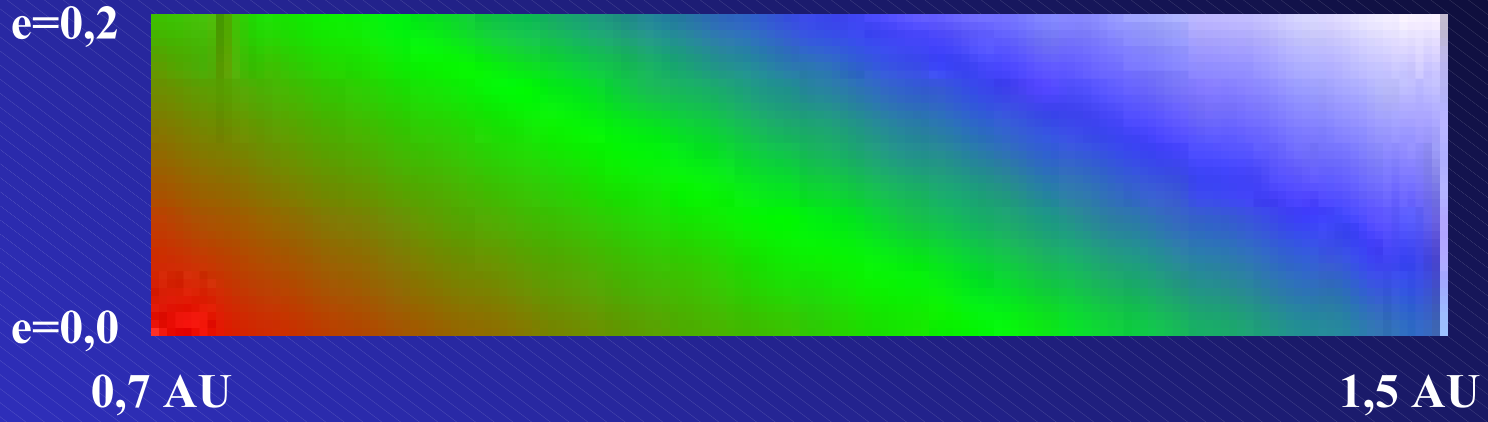
- Parameters:
  - mass of the star: 1,062 solar mass
  - one planet: m = 4,0 M_{Jup}, a = 3,2 AU, e = 0,45
- The stable regions are very narrow in the habitable zone, the chaotic strips of the resonances penetrate deeply

ε Eridani:



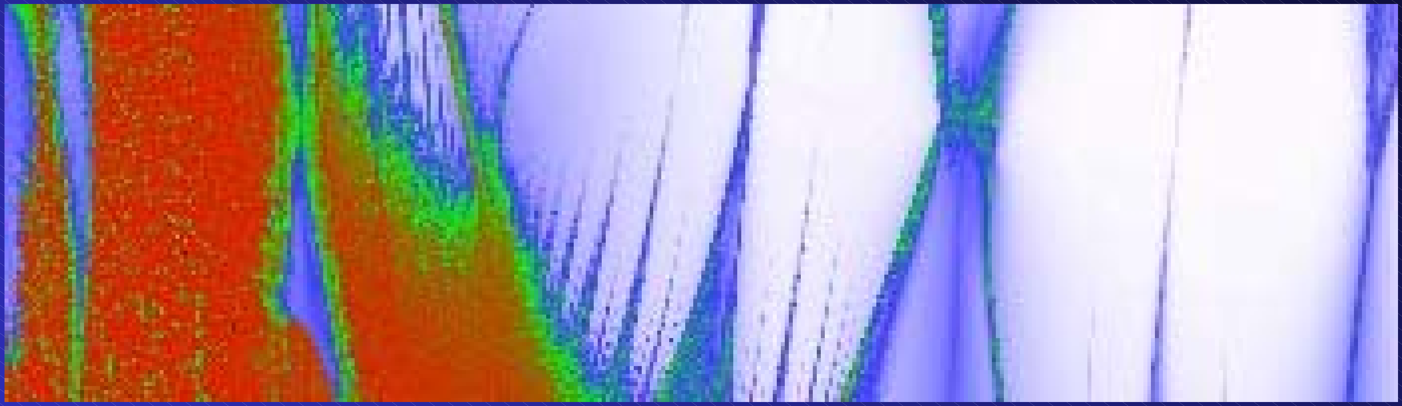
- **Parameters:**
  - **mass of the star : 0,798 solar mass**
  - **one planet: m = 0,9 M_{Jup}, a = 3,36 AU, e = 0,60**
- **The inner part of the habitable zone seems to be stable, but there are high-order resonances. These resonances do not disappear by increasing the inclination.**

ρ Coronae Borealis:



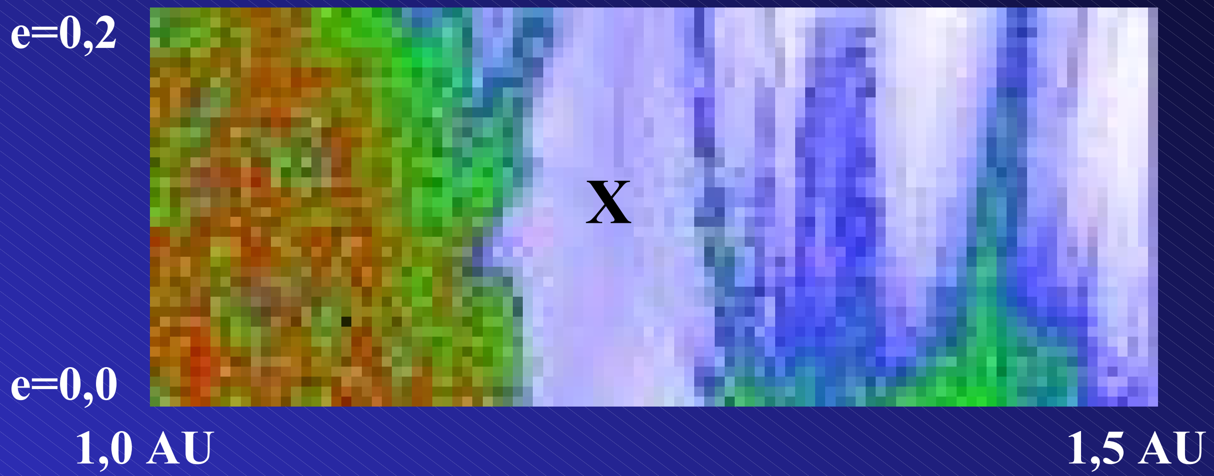
- **Parameters:**
  - mass of the star: 0,946 solar mass
  - one planet: m = 1,3 M_{Jup}, a = 0,224 AU, e = 0,07
- **The habitable zone is stable**

70 Virginis:



- **Parameters:**
  - mass of the star: 1,097 solar mass
  - one planet: $m = 7{,}4\ M_{Jup}$, $a = 0{,}482$ AU, $e = 0{,}40$
- **The outer part of the habitable zone is stable**

HD 80606:



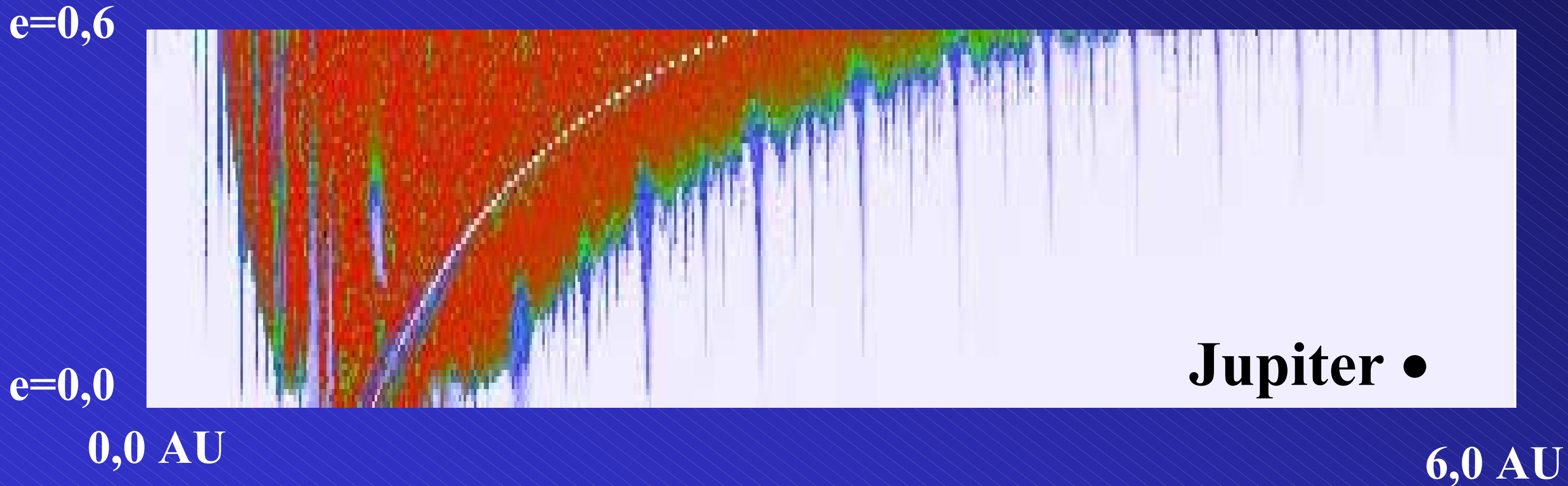
- **Parameters:**
  - mass of the star: 0,817 solar mass
  - one planet: m = 3,4 M_{Jup}, a = 0,438 AU, e = 0,93
- **The inner part of the habitable zone is not stable (the eccentricity of the perturbing planet is large), the outer part is stable at large eccentricity**
- **Critical areas (eg. marked with 'X'): the long-time numerical integration of show that this region of the phase space is not stable**

47 Ursae Maioris:

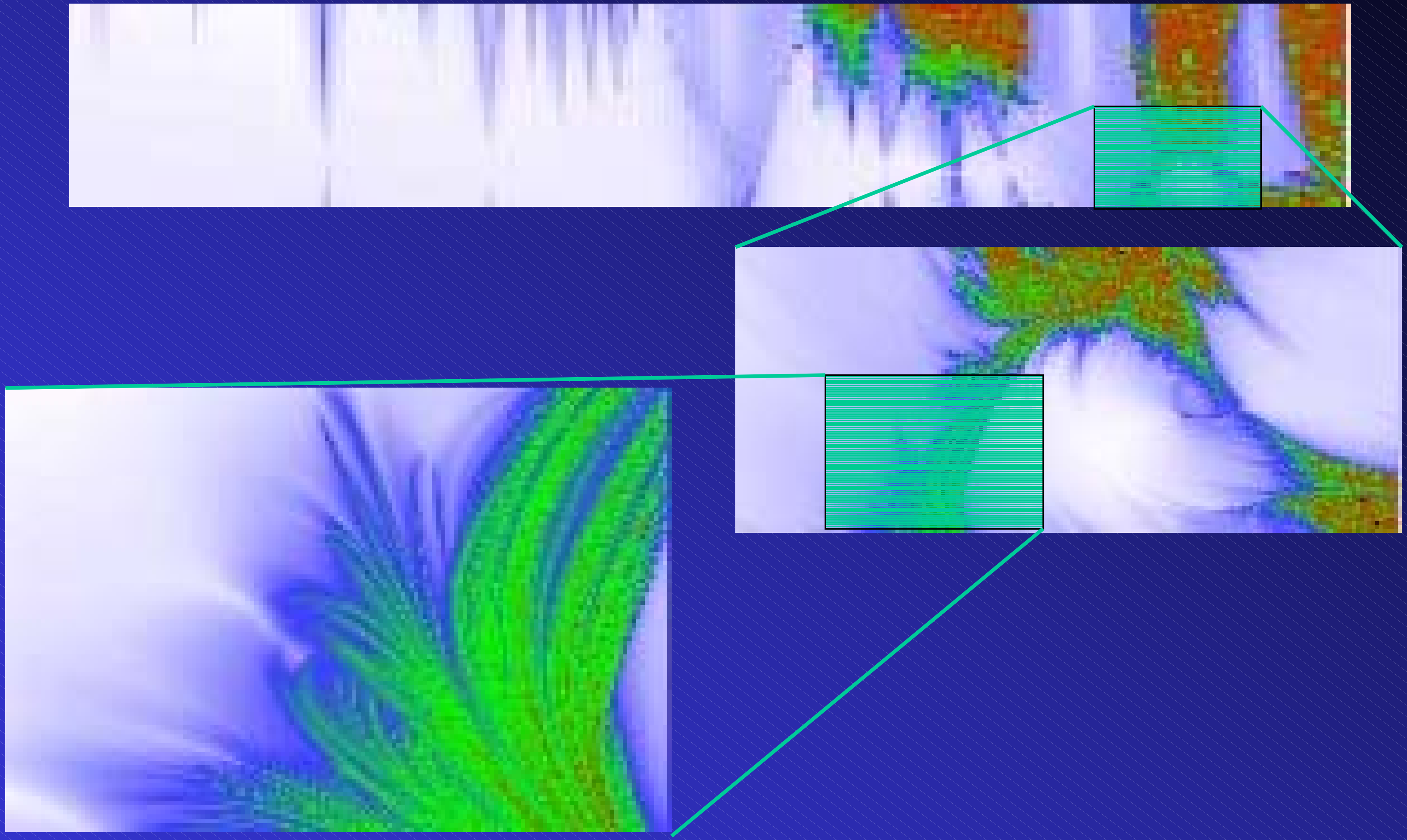
- Parameters:
  - mass of the star: 1,03 solar mass
  - two planets - this system is dynamically similar to our Solar System
- The habitable zone is stable except the neighbourhood of the 2:1 and 3:1 resonances

Other approach:

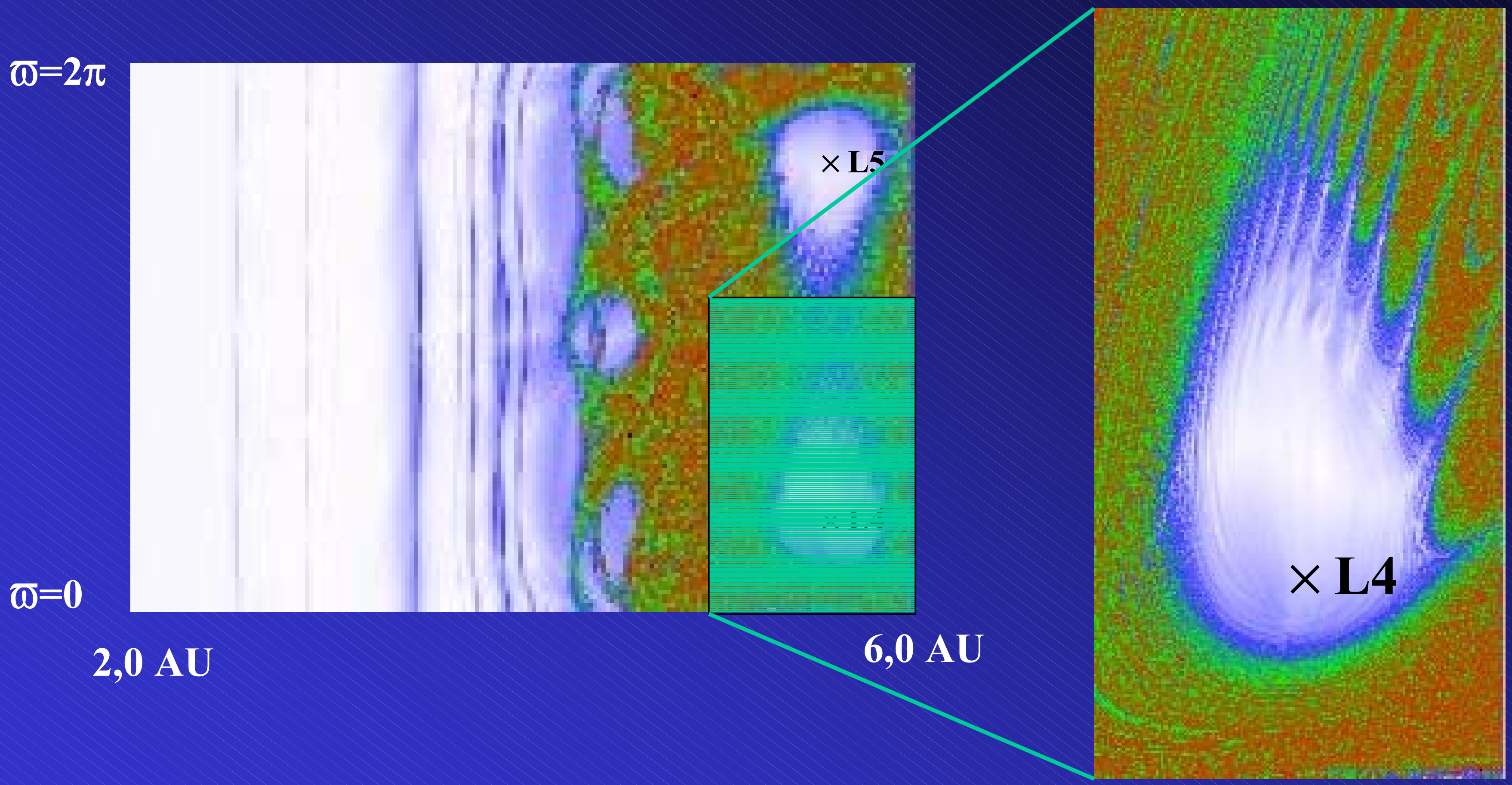
- The phase space can be examined by changing the orbital elements (a, e) of the perturbing planet
- The investigated systems can be plotted in these diagrams



Our planetary sytem:



Trojans:



Summary

- We applied the method of finite-time relative Lyapunov indicators efficiently to study the stability regions in exoplanetary systems
- Our investigations gave the same results as the previous examinations
- Investigations in three dimension: calculate the case when the inclination changes (ε Eridani)