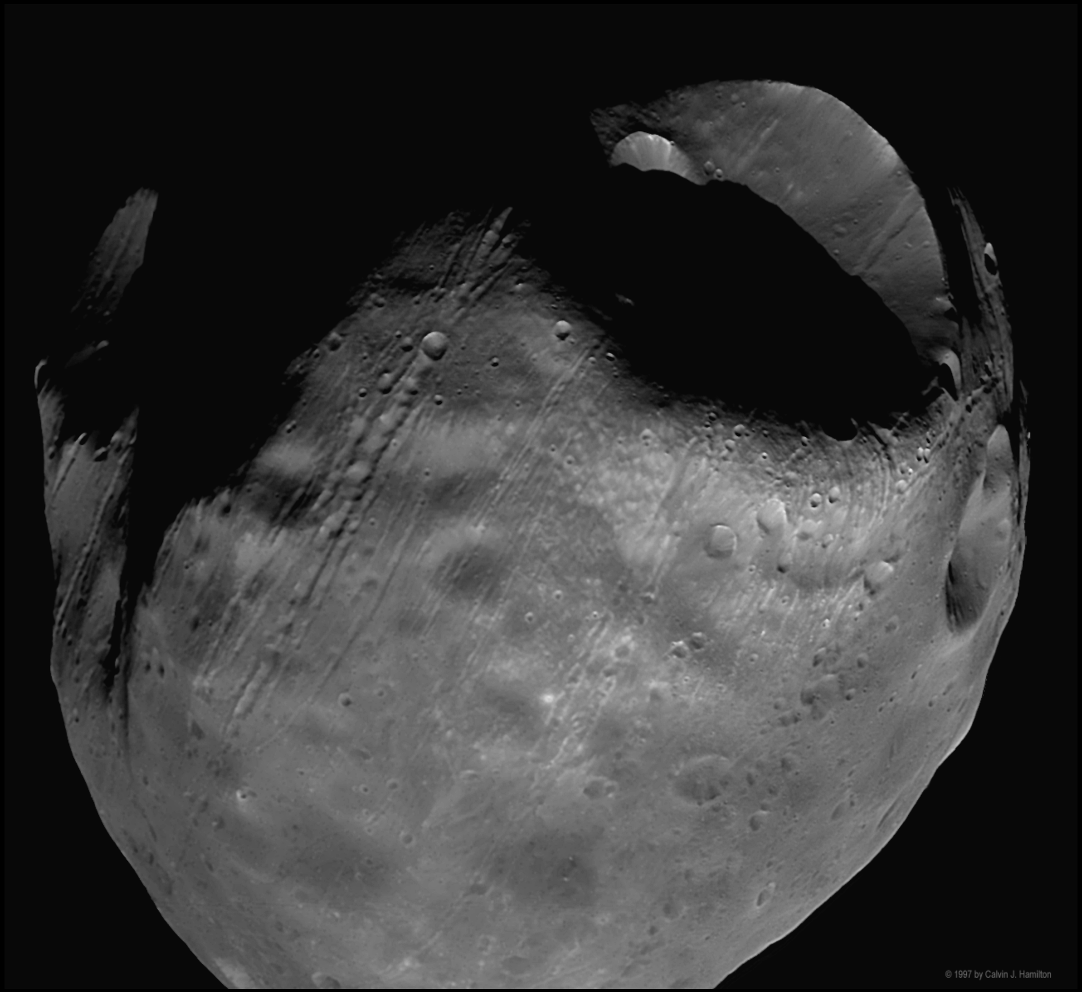
The Rotation of Natural Satellites



3th Austrian-Hungarian Workshop

on Trojans and related Topics

Spin influenced by

- Wobble decay
- Tides
- Spin orbit coupling

Wobble decay:

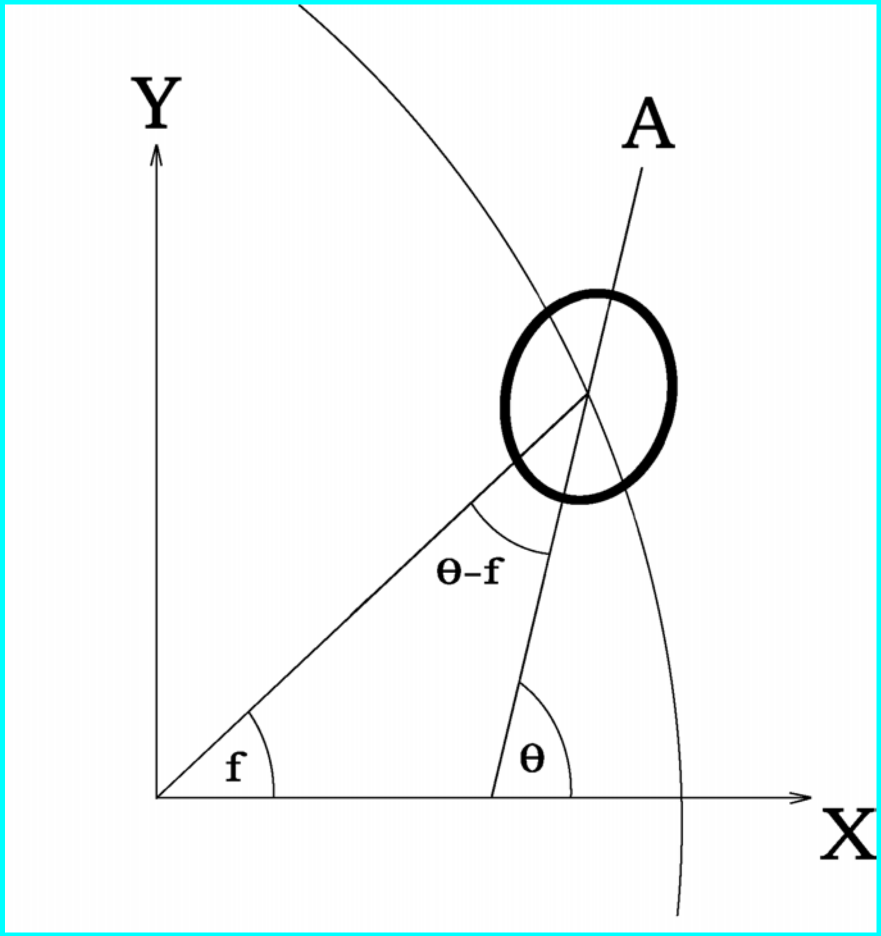
- Periodic distortion of the body rotation of the satellite – > principal axis rotation
- Time scale: small

Tides:

- spin-angular momentum $->$ orbit
- gravitational field distorts the satellite

  phase lag δ in the response of grav. field
- Ocean tides
- solid body tides

Definitions



Edge-on-view of the satellites‘ orbit

$$\frac{d^2\theta}{dt^2} + \frac{3}{2}\frac{B-A}{C}n^2\left(\frac{a}{r}\right)^3 \sin(2(\theta - f)) = W$$

W	tidal torque
a	orbital semimajor axis
r	distance between the planet and the satellite
A, B, C	principal axes
n	orbital mean motion
f	true anomaly

Circular orbit

- Angular Momentum

  satellites spin $-$> orbit

  synchronous rotation

Non circular orbit

- Tidal Torque W

Time rate of changing the spin angular velocity

$$W = \frac{d\omega}{dt} = -\frac{3}{2}\frac{k_2 G M^2 R^5}{r^6}\sin(2\delta) = -\frac{45\rho R^4 n^4}{38\mu Q}\left(\frac{a}{r}\right)^6$$

$\frac{d\theta}{dt} = \omega$	spin angular velocity
M	planetary mass
R	mean equatorial radius
r	distance between the planet and the satellite
μ	rigidy of the satellite
ρ	mass density
a	orbital semimajor axis
Q	specific dissipation function

$$\frac{1}{Q} = \frac{1}{2\pi E^*}\oint \frac{dE}{dt}\,dt \qquad \frac{1}{Q} \approx 2\delta$$

E^*	peak energy stored in an oscillating system
$\oint \frac{dE}{dt}\,dt$	energy dissipated over a cycle of oscillation

Spin orbit coupling:

- End point of tidal evolution

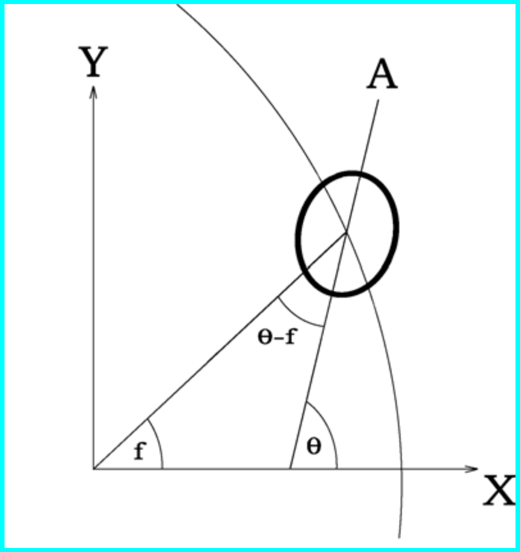
  $\langle W \rangle = 0$
- non spherical shaped satellites

  $->$ resonant torque (caused by spin orbit coupling) possible

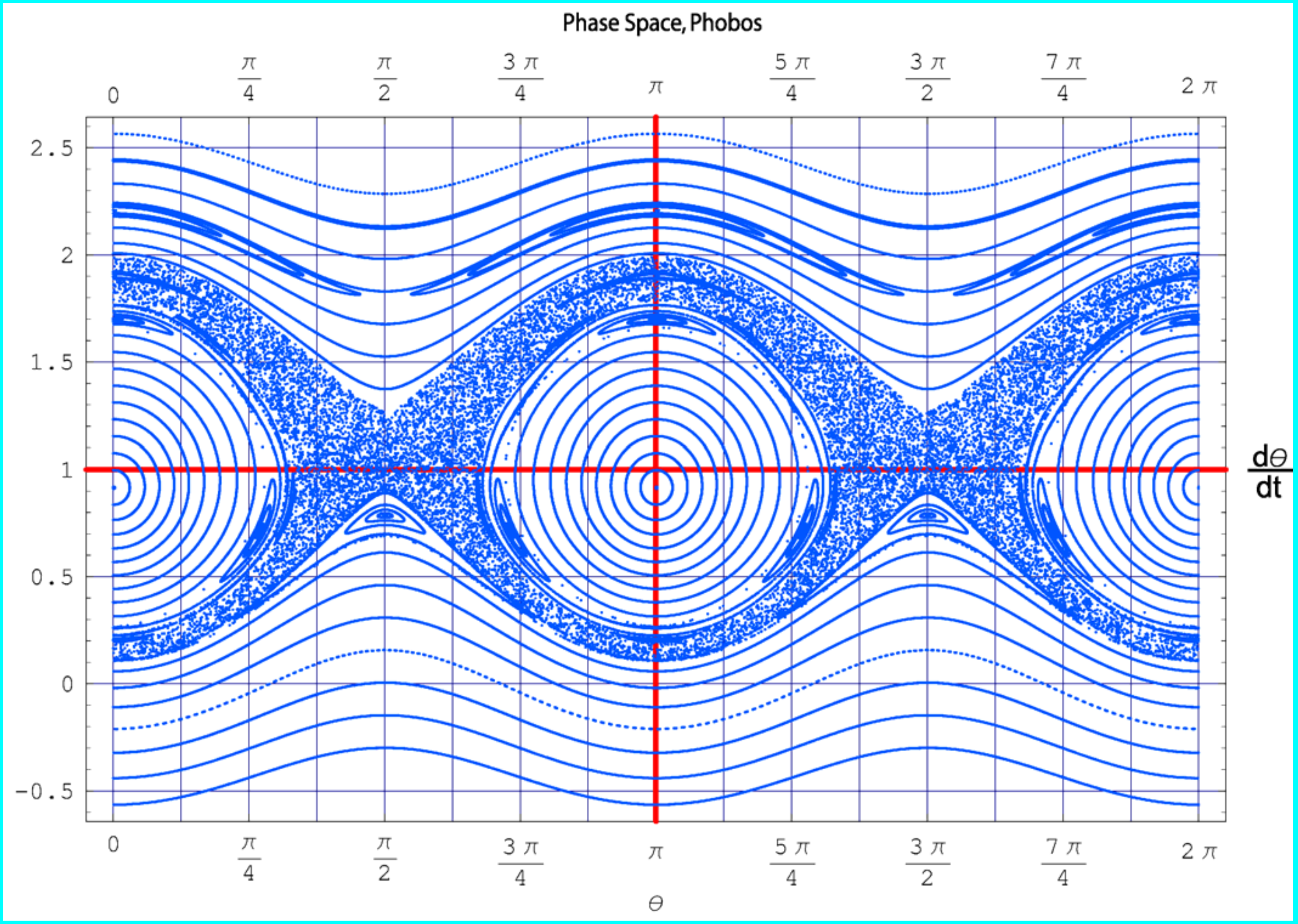
  $\frac{d\theta}{dt} = pn \qquad n = \frac{2\pi}{T}$

Phobos - Deimos

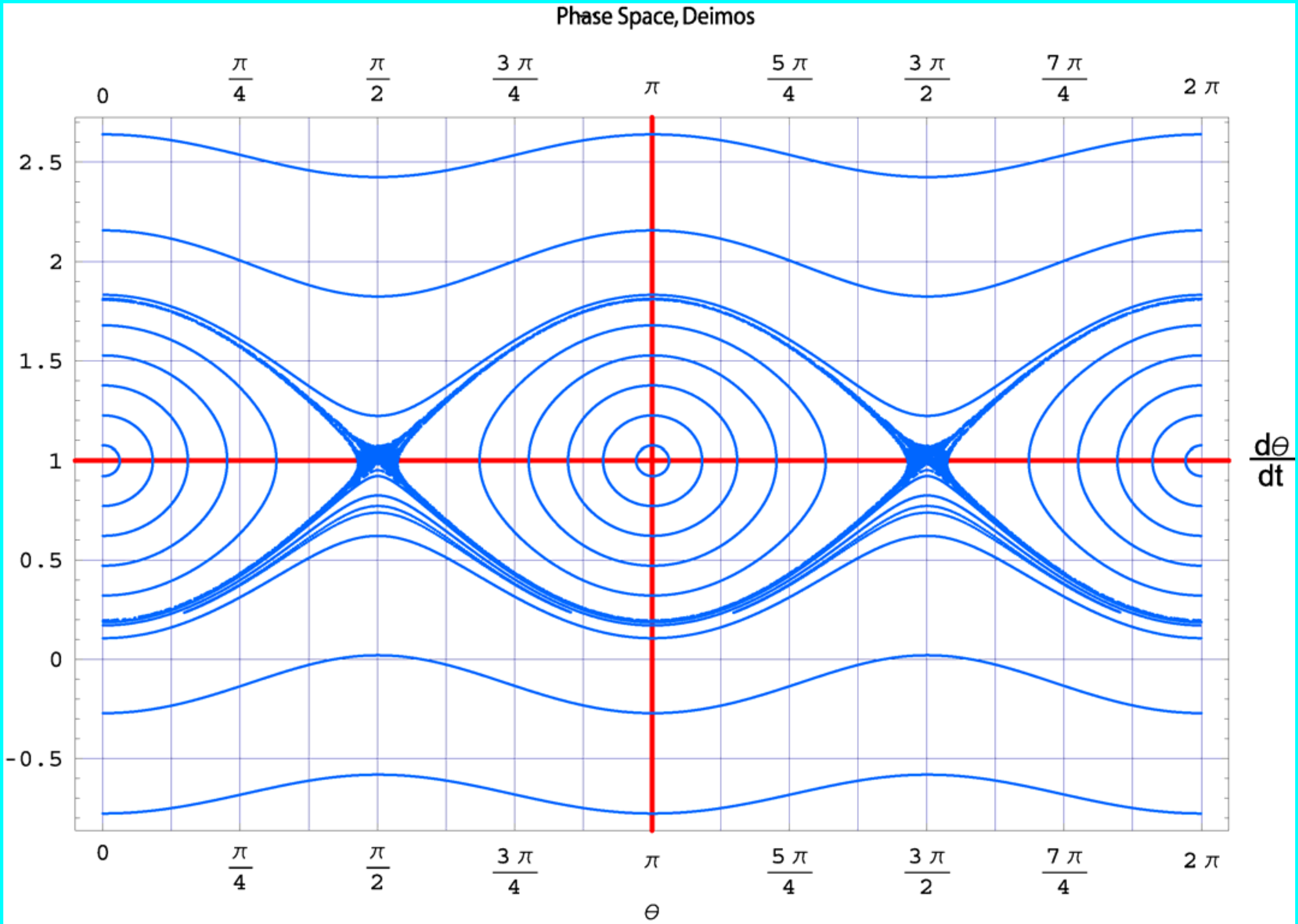
- Equation $\frac{d^2\theta}{dt^2}+\frac{3}{2}\frac{B-A}{C}n^2\left(\frac{a}{r}\right)^3\sin(2(\theta-f))=W$ for W=0 was solved.
- Runge Kutta 4th order
- Program generator for Mathematica execution $-\!>$ Plots: spin angular velocity $\frac{d\theta}{dt}$ vs. θ at perihelion



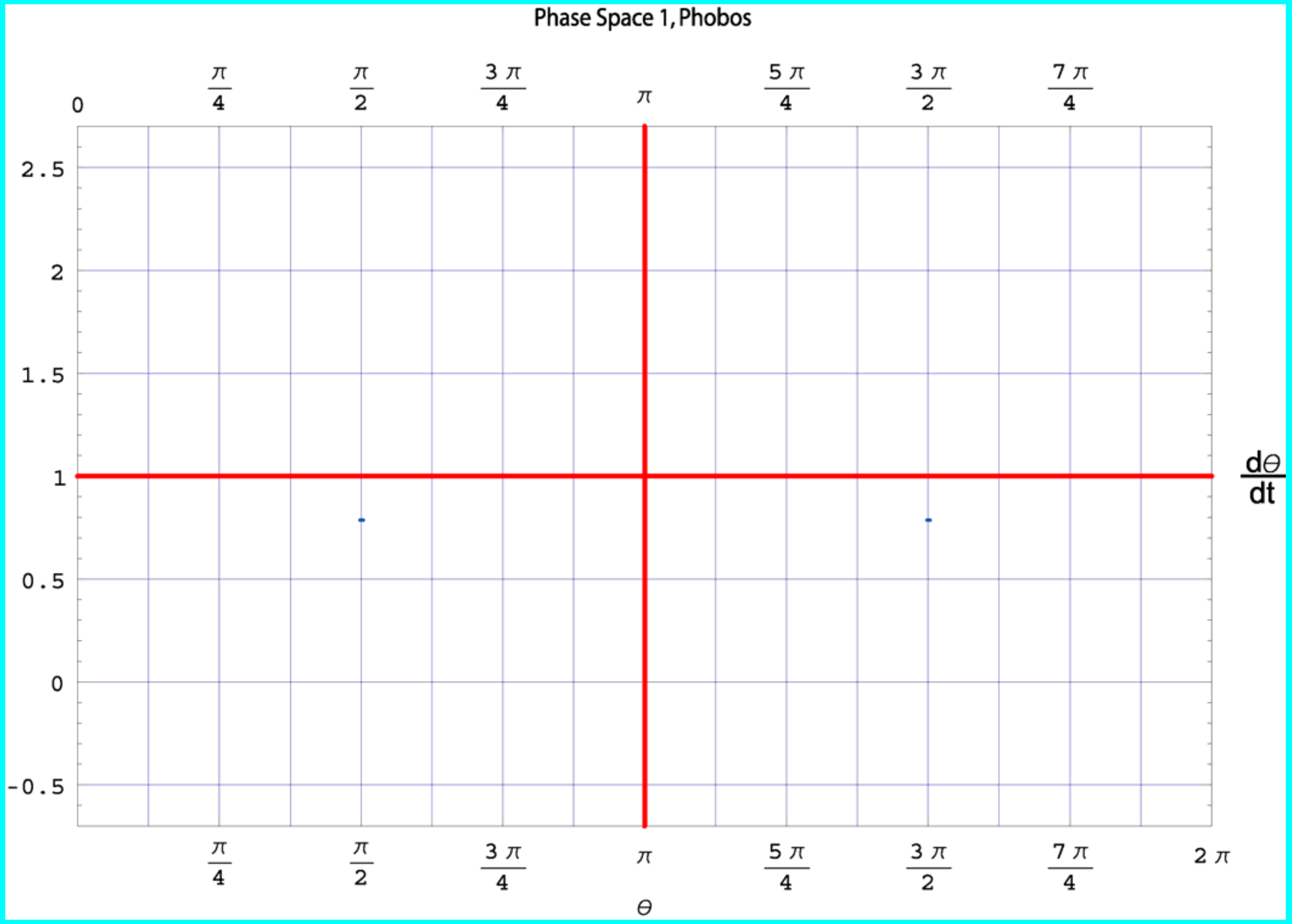
- For some special initial conditions:
  limb profiles at perihelion
  frequency analysis (FFT with 2^{20} Points)
  Lyapunov Exponents



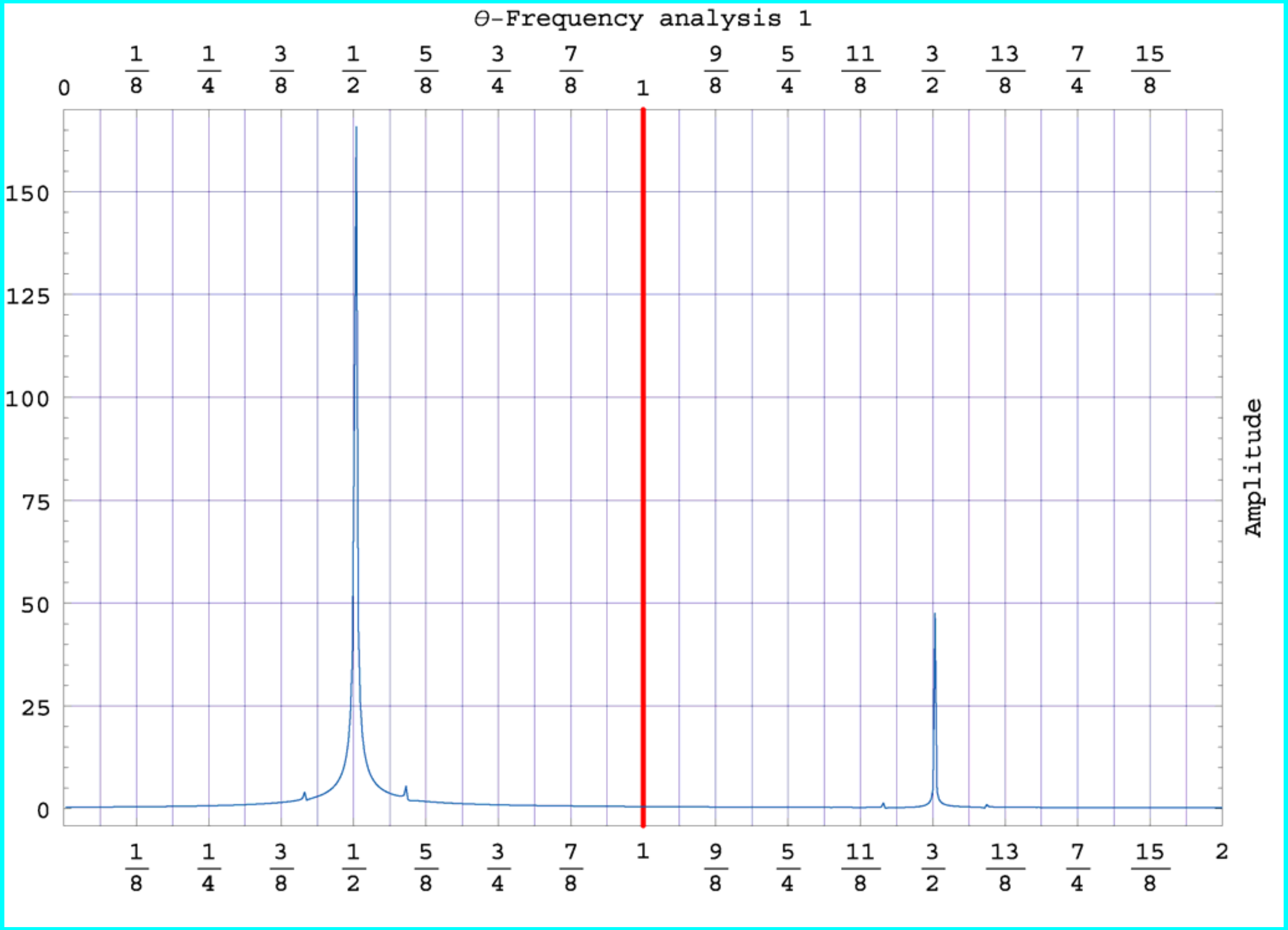
Phase Space, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ

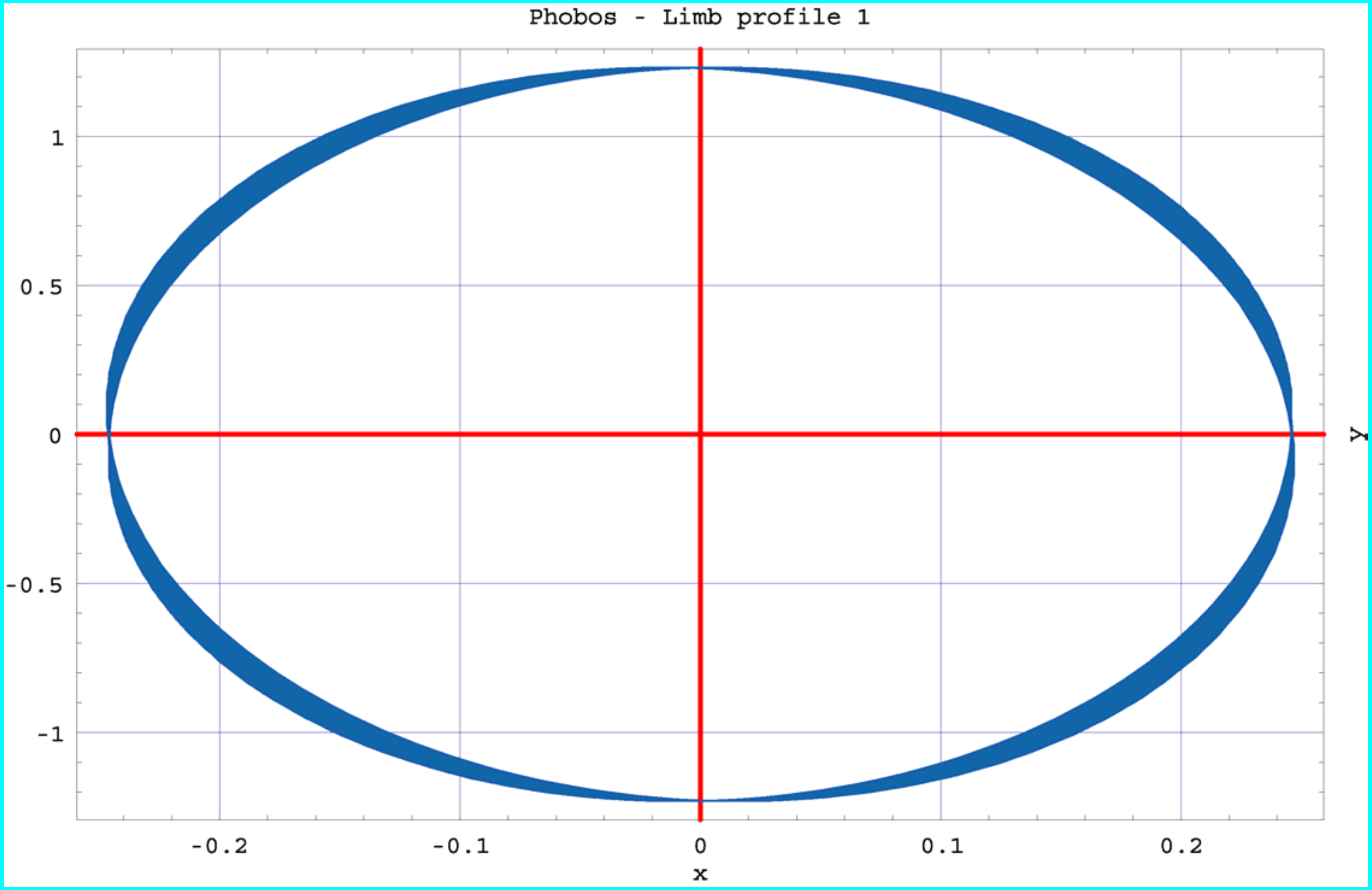


Phase Space, Deimos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ

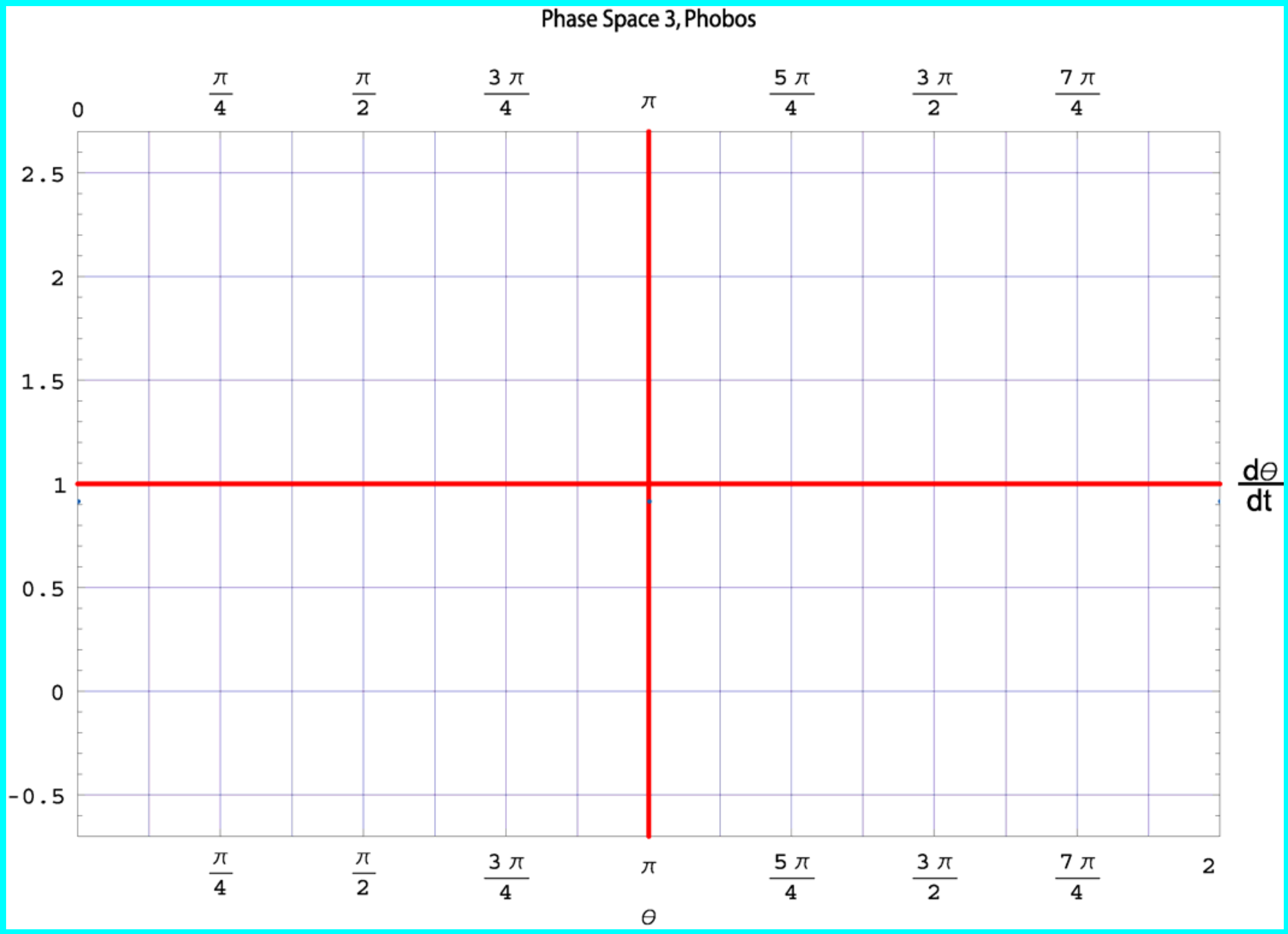


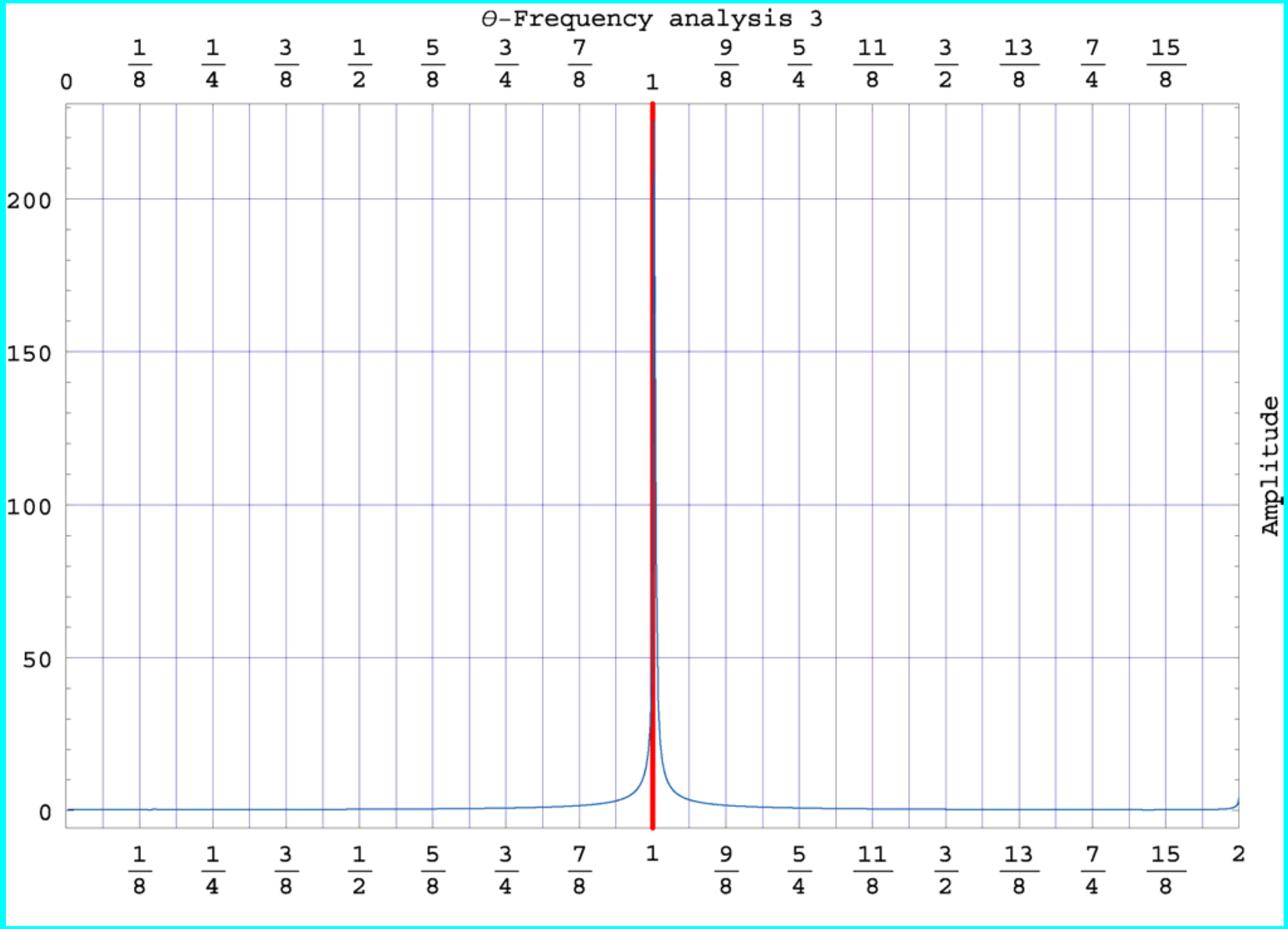
Phase Space 1, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ


θ-Frequency analysis 1
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
9/8
5/4
11/8
3/2
13/8
7/4
15/8
2
150
125
100
75
50
25
0
Amplitude

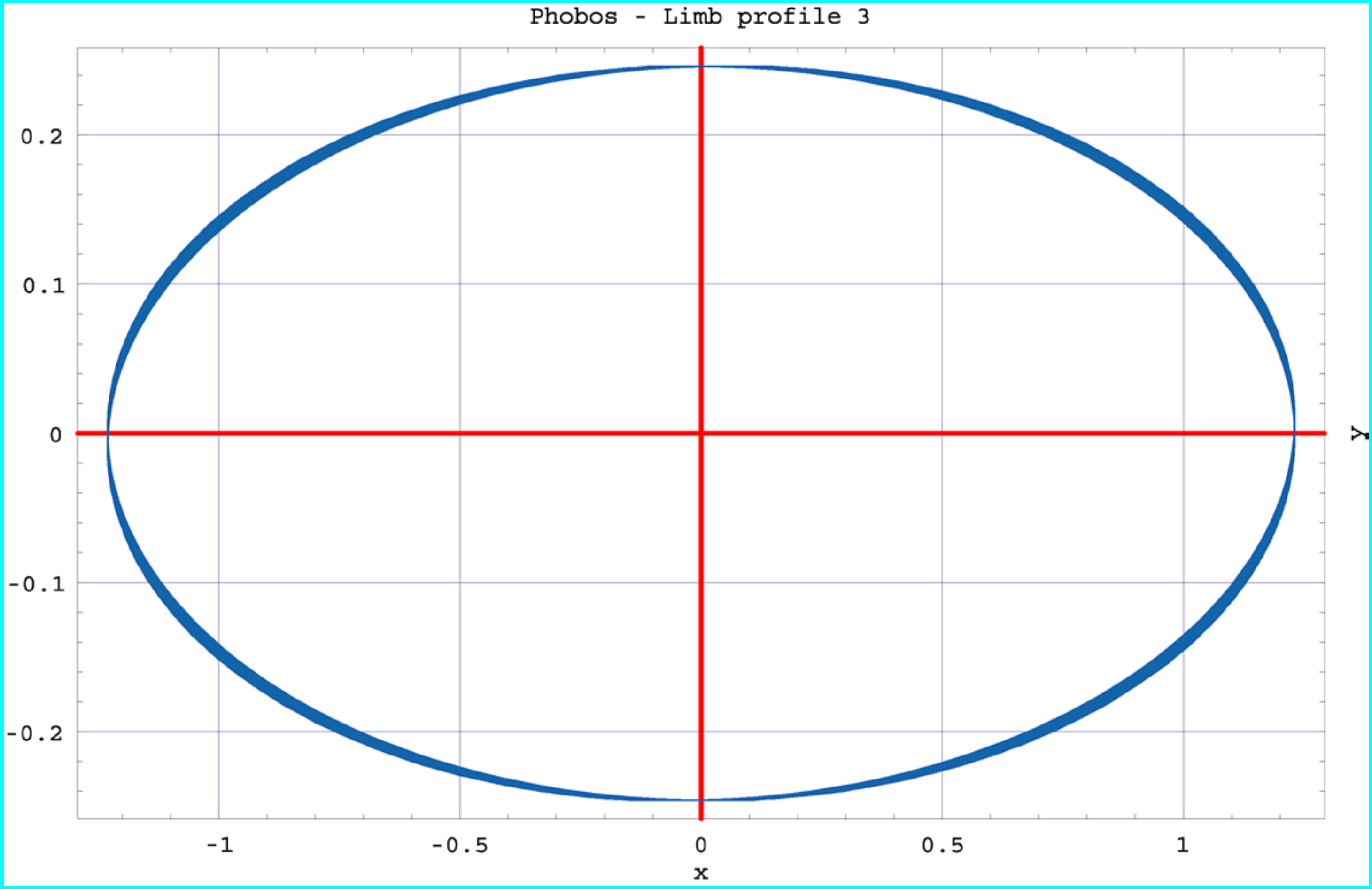


Phobos - Limb profile 1
1
0.5
0
-0.5
-1
-0.2
-0.1
0
0.1
0.2
x
y

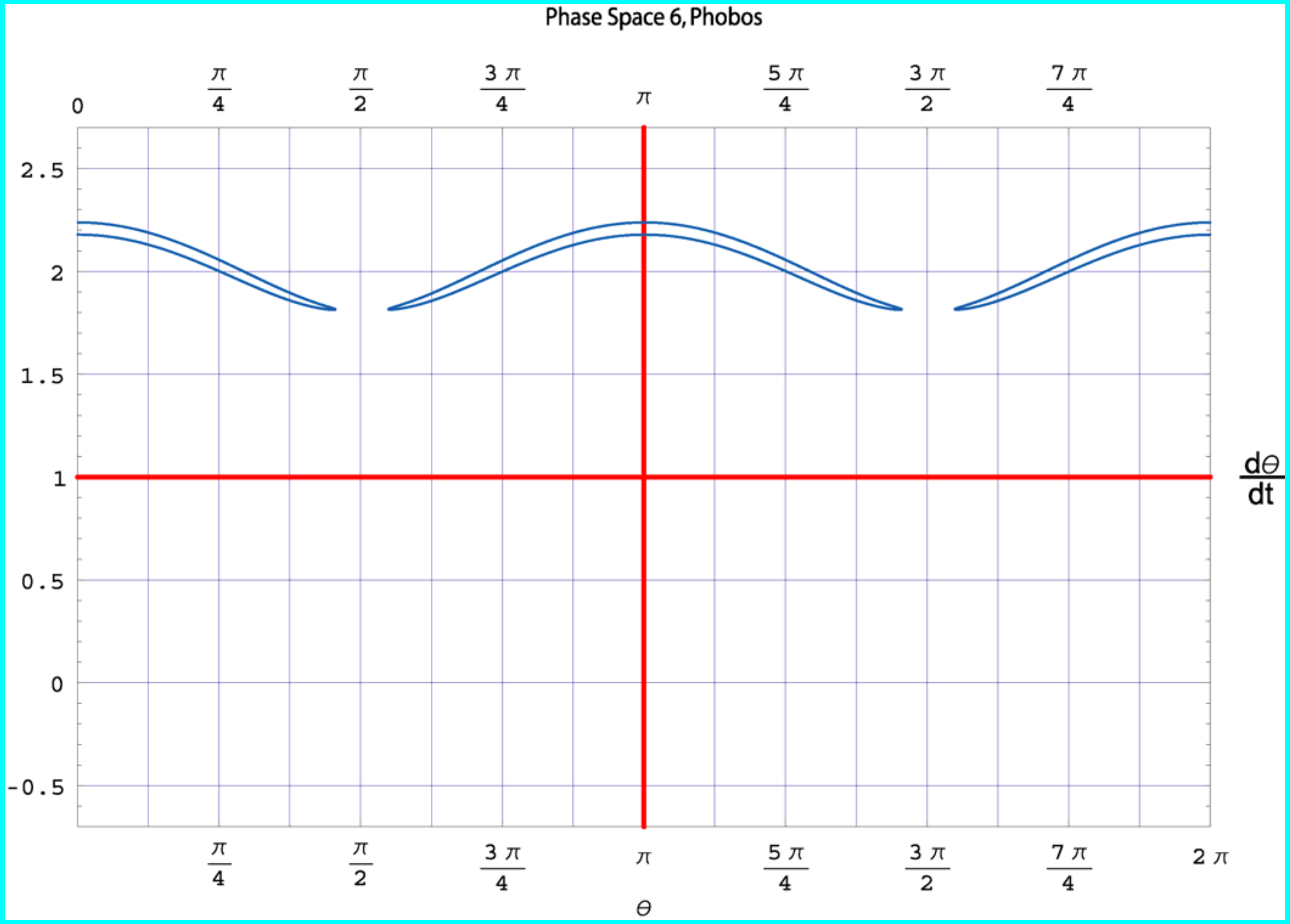

Phase Space 3, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ



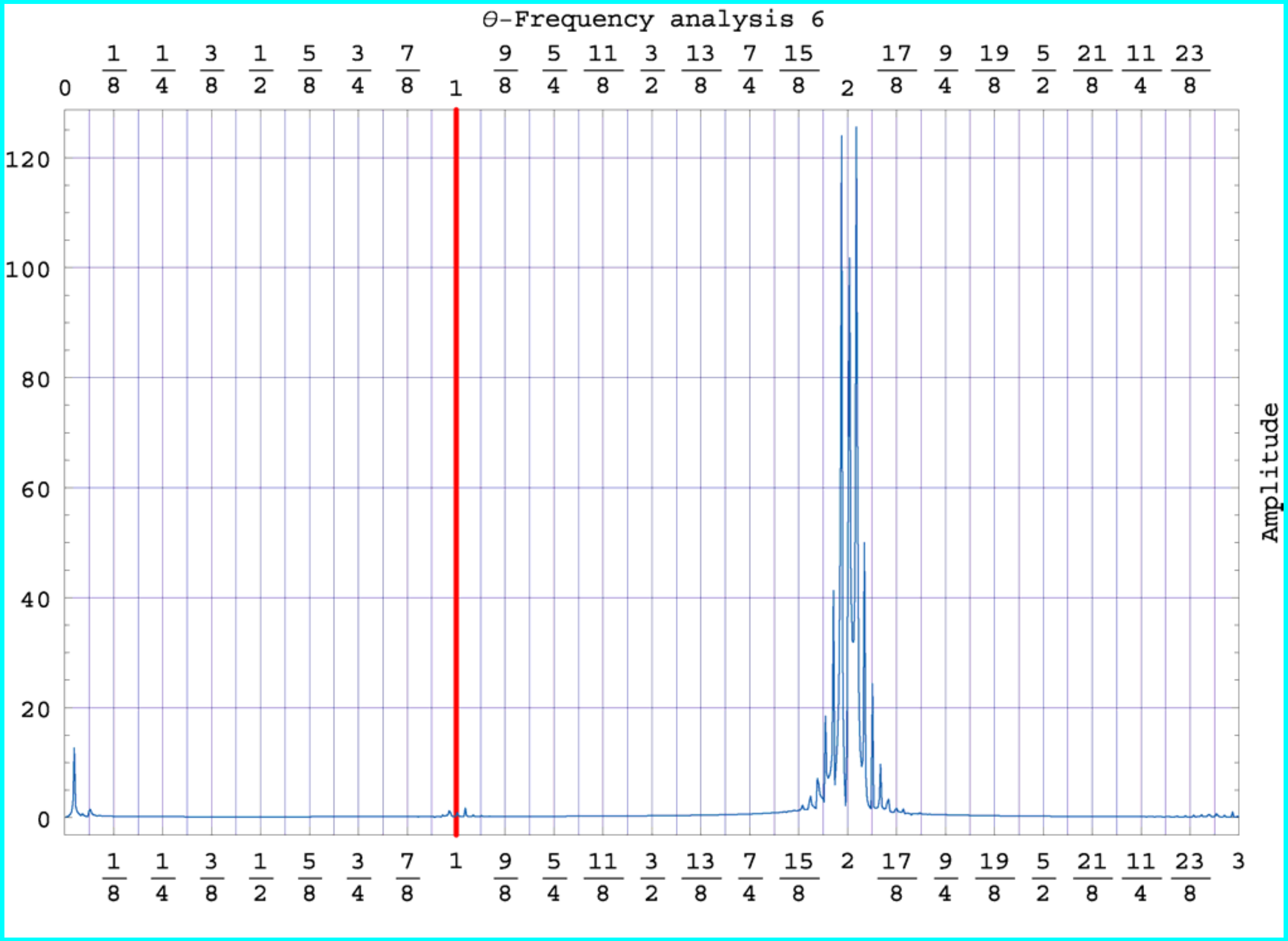
θ-Frequency analysis 3
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
9/8
5/4
11/8
3/2
13/8
7/4
15/8
2
200
150
100
50
0
Amplitude

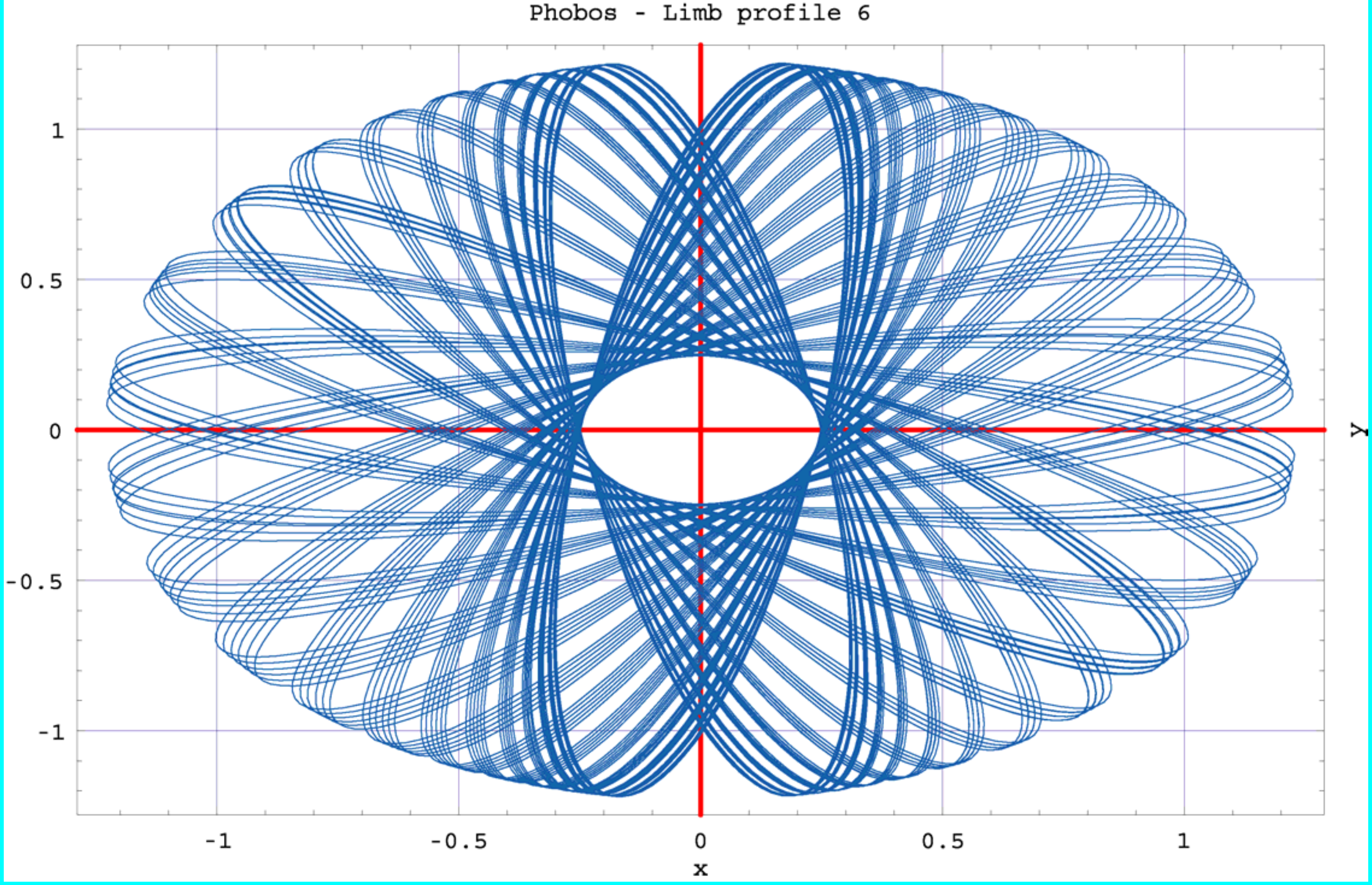


Phobos - Limb profile 3
0.2
0.1
0
-0.1
-0.2
-1
-0.5
0
0.5
1
x
y

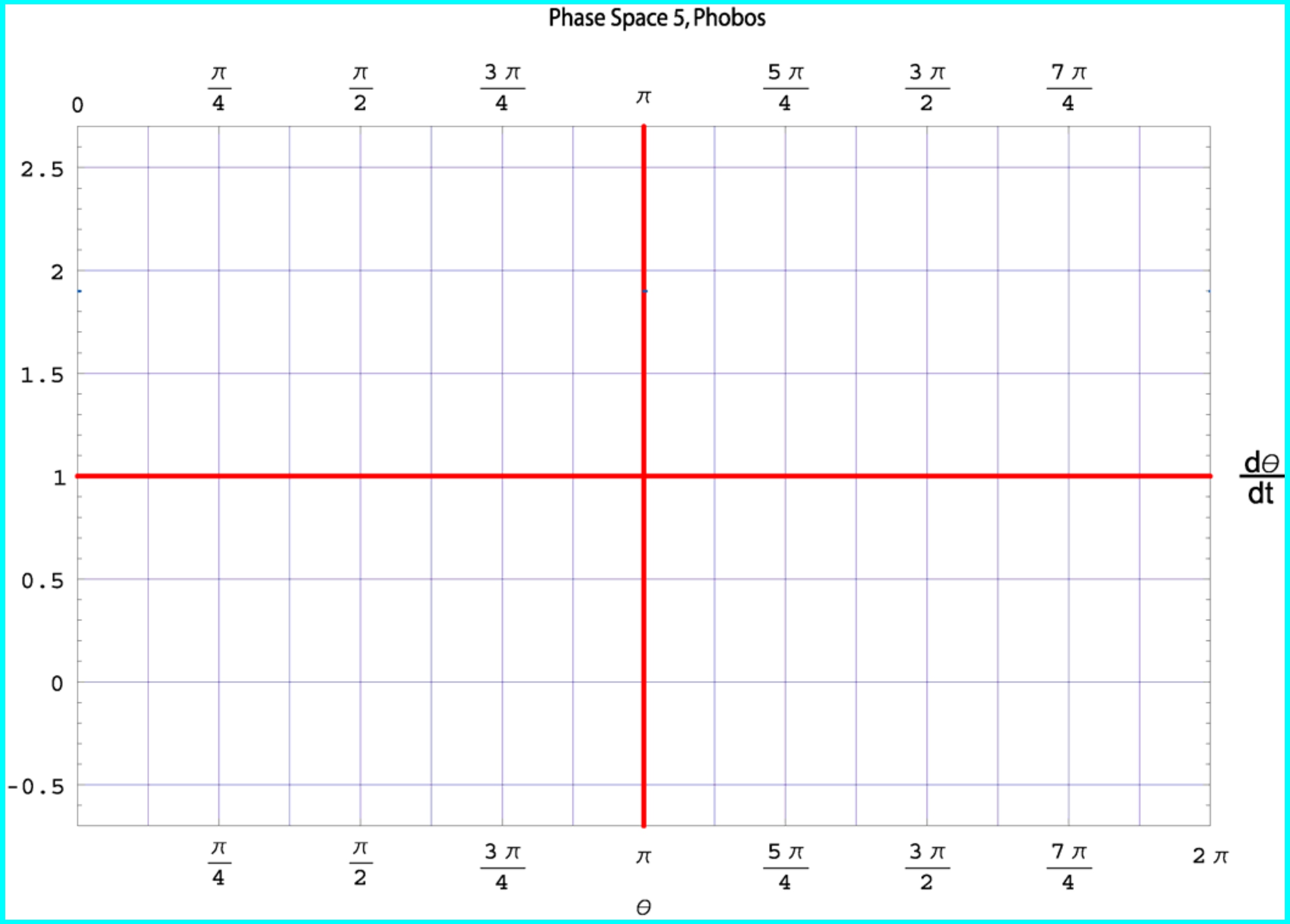


Phase Space 6, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ

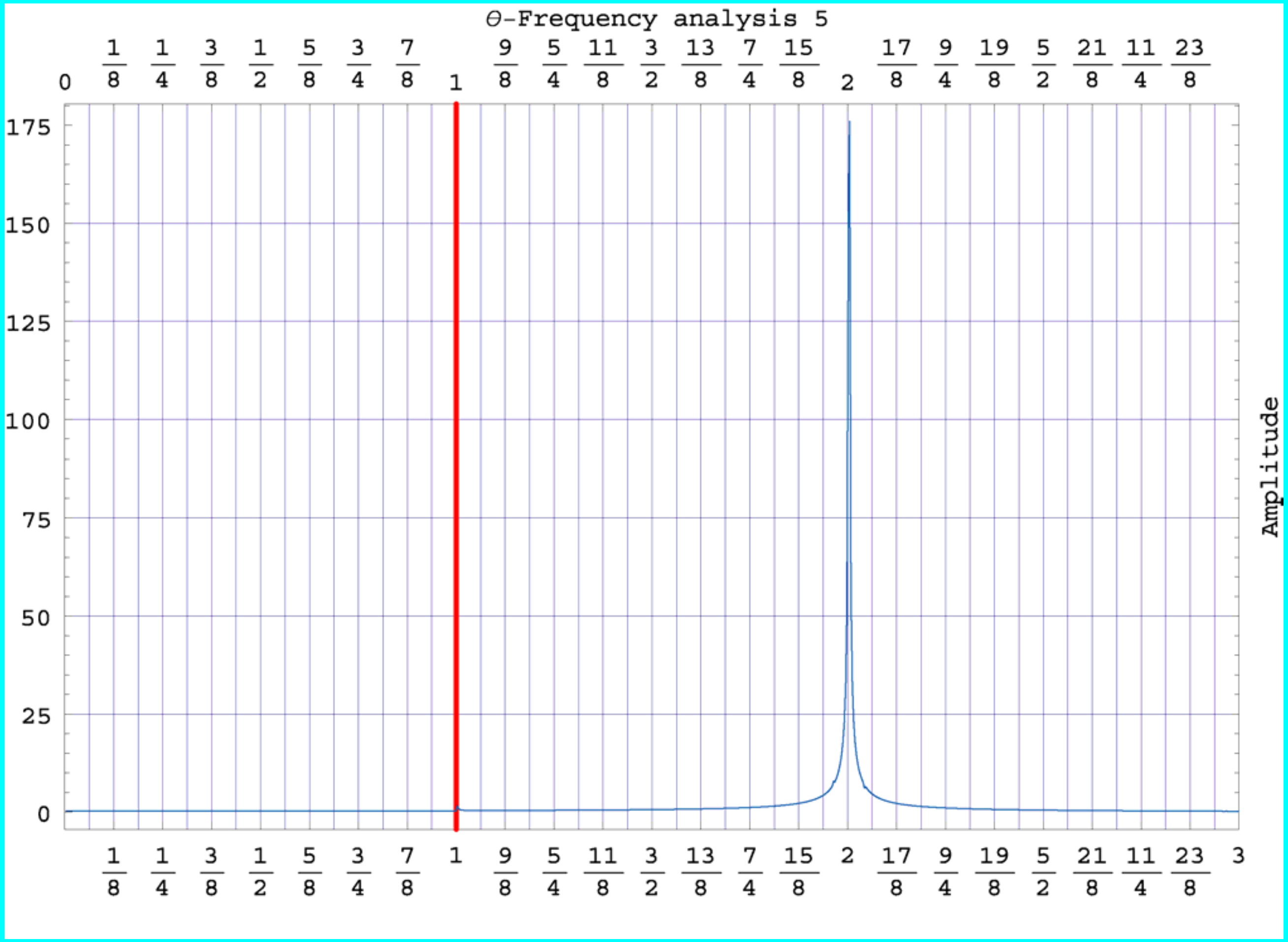

θ-Frequency analysis 6
Amplitude
0
20
40
60
80
100
120
1/8 1/4 3/8 1/2 5/8 3/4 7/8 1 9/8 5/4 11/8 3/2 13/8 7/4 15/8 2 17/8 9/4 19/8 5/2 21/8 11/4 23/8 3



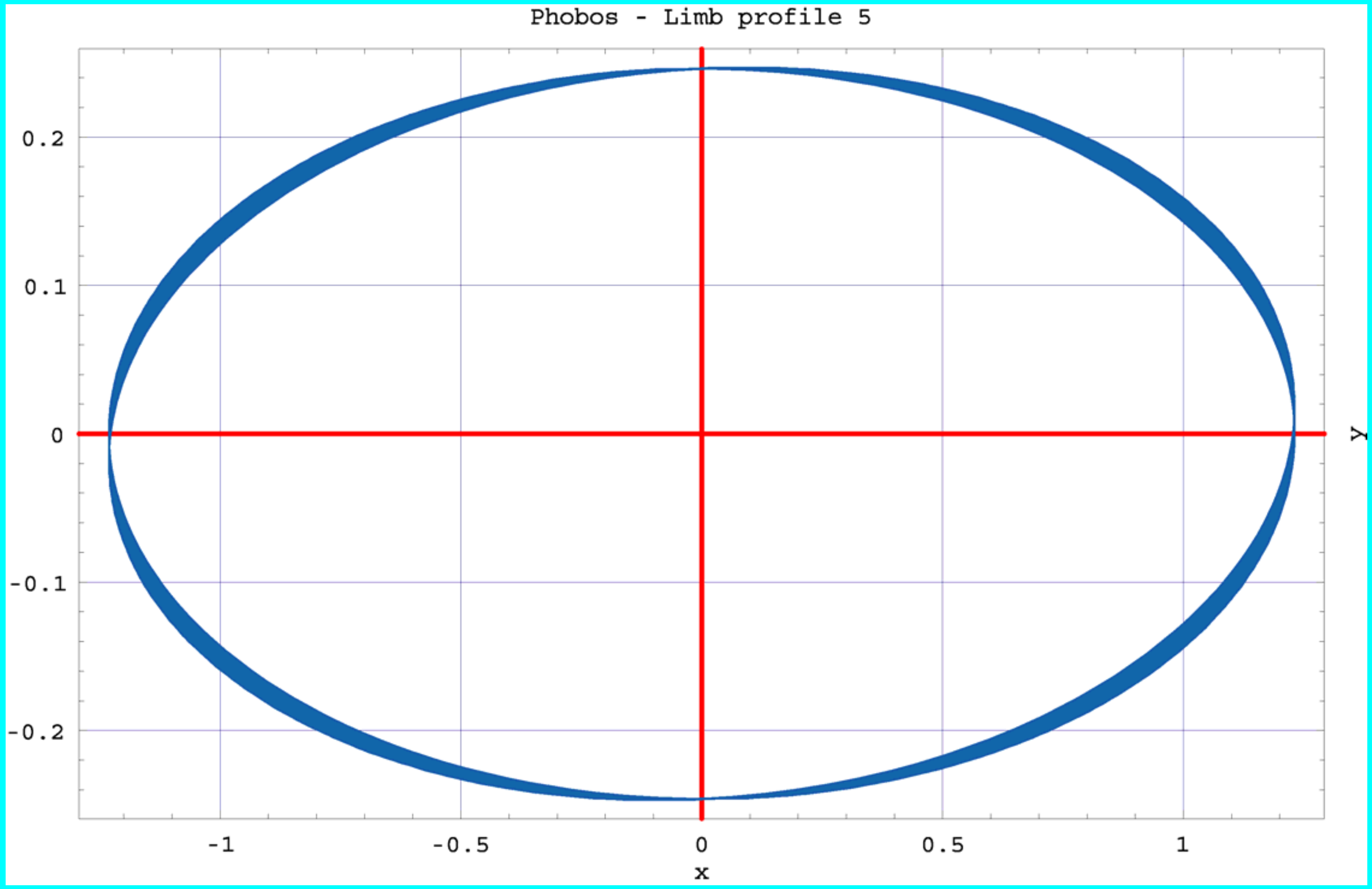
Phobos - Limb profile 6
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
x
y



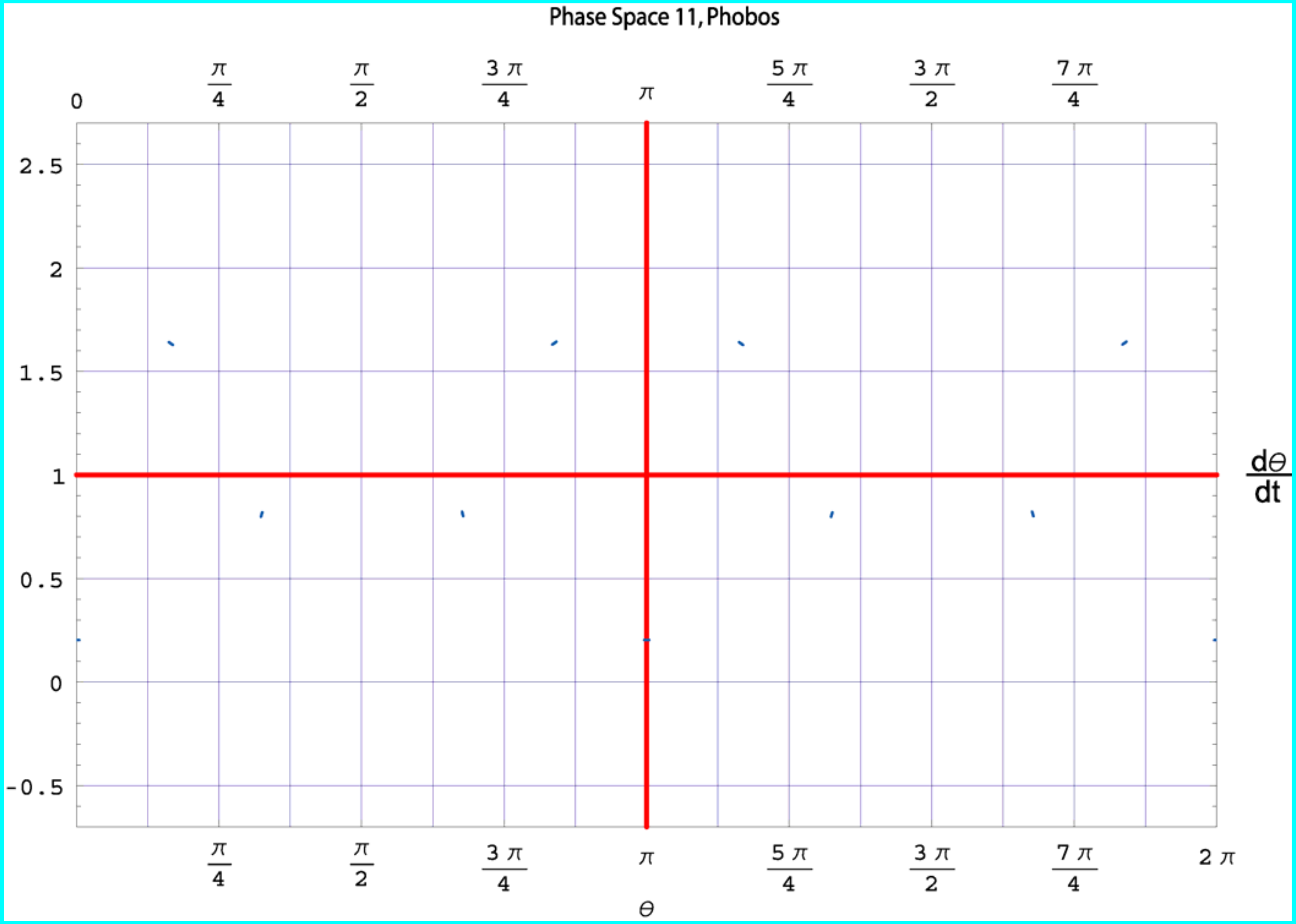
Phase Space 5, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ



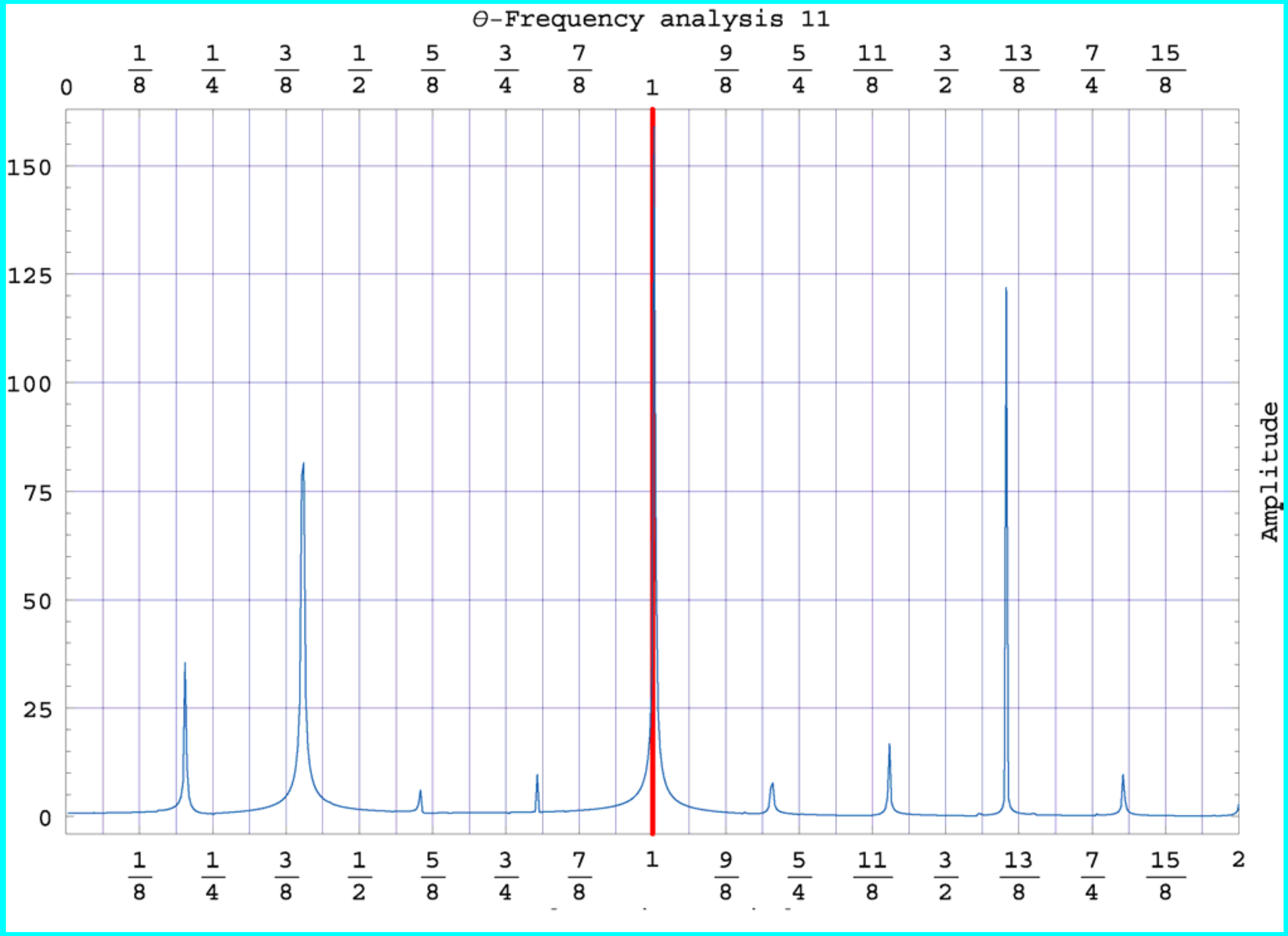
Θ-Frequency analysis 5
Amplitude
0
25
50
75
100
125
150
175
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
9/8
5/4
11/8
3/2
13/8
7/4
15/8
2
17/8
9/4
19/8
5/2
21/8
11/4
23/8
3



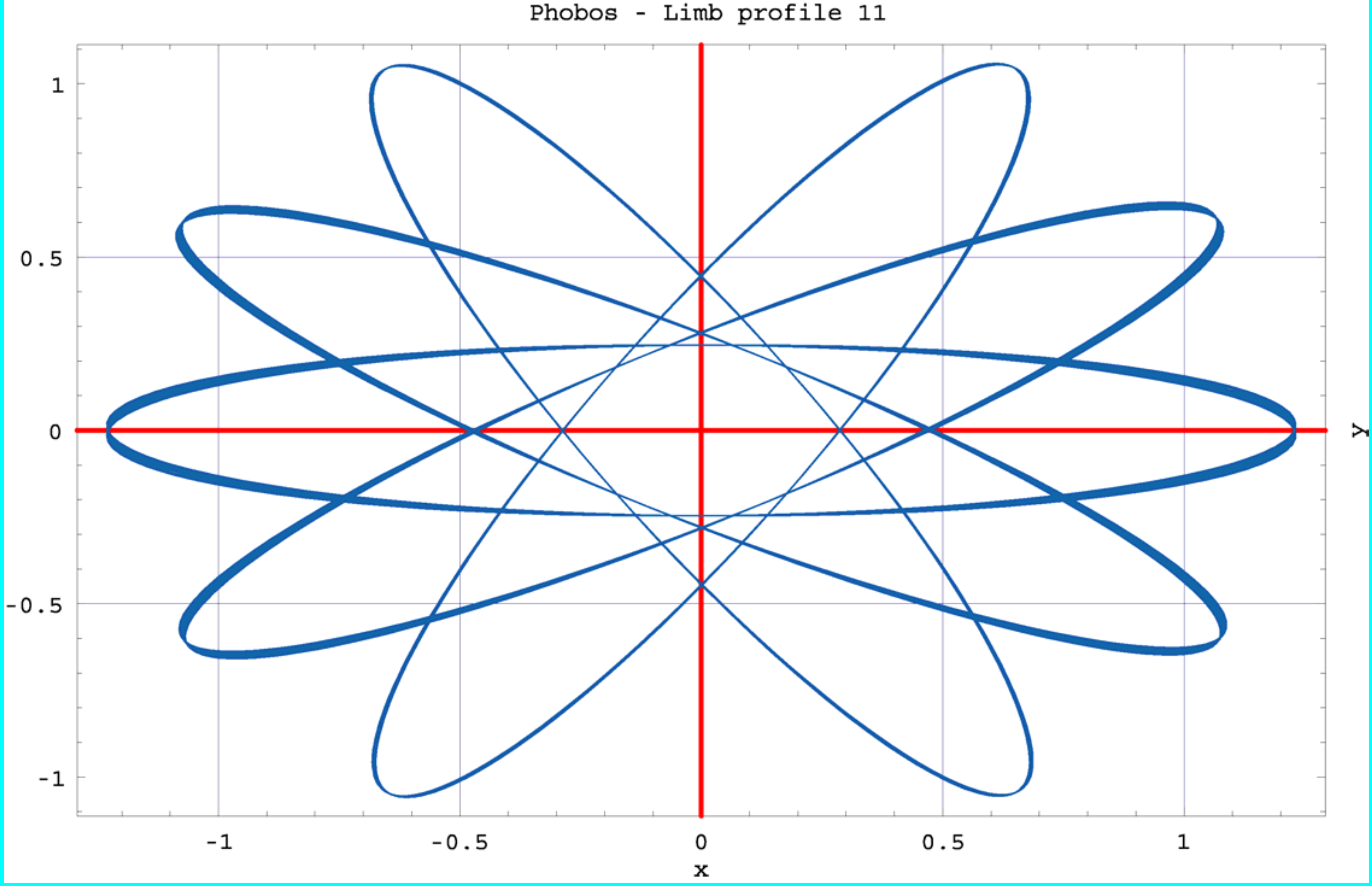
Phobos - Limb profile 5
0.2
0.1
0
-0.1
-0.2
-1
-0.5
0
0.5
1
x
y



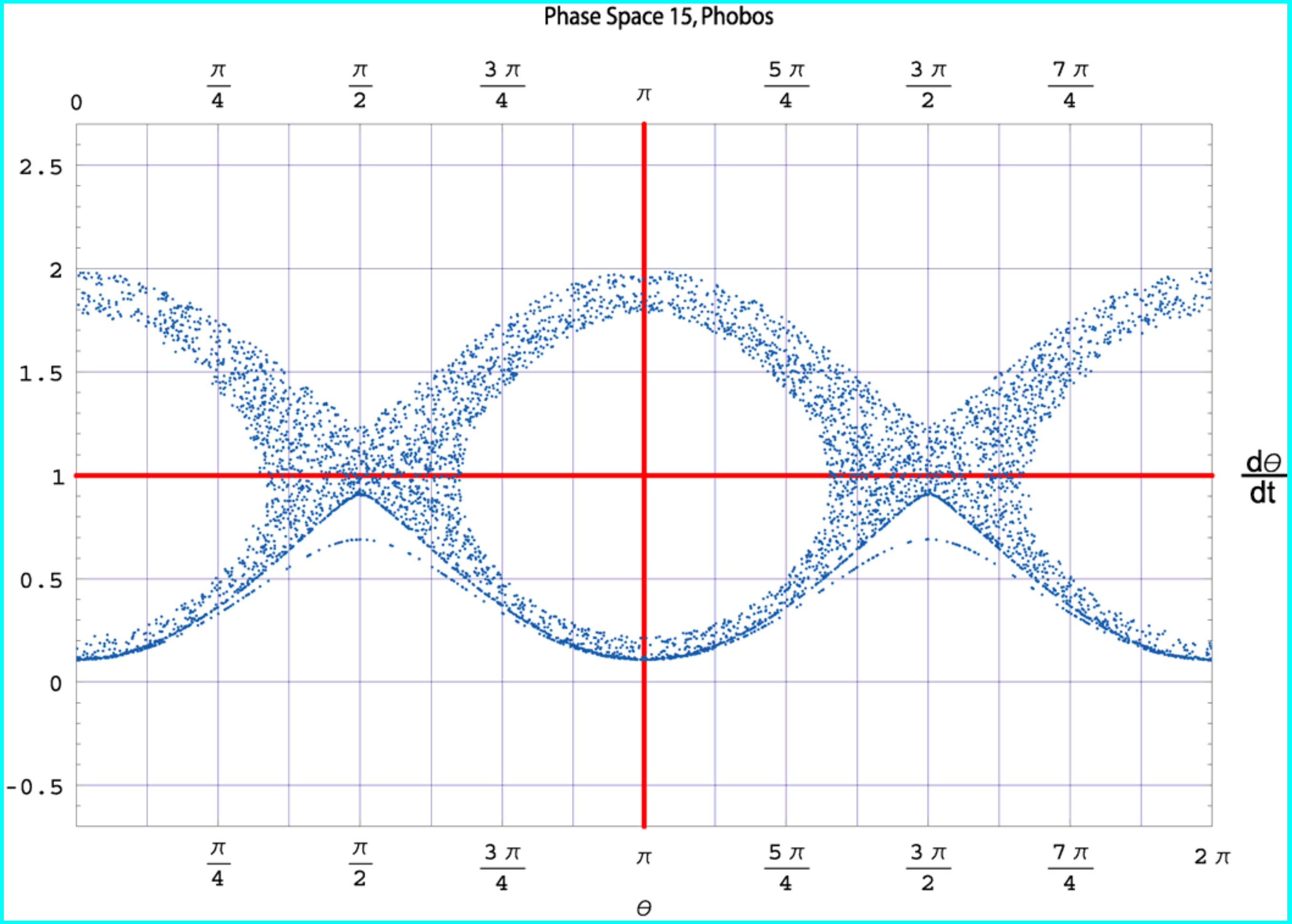
Phase Space 11, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
θ



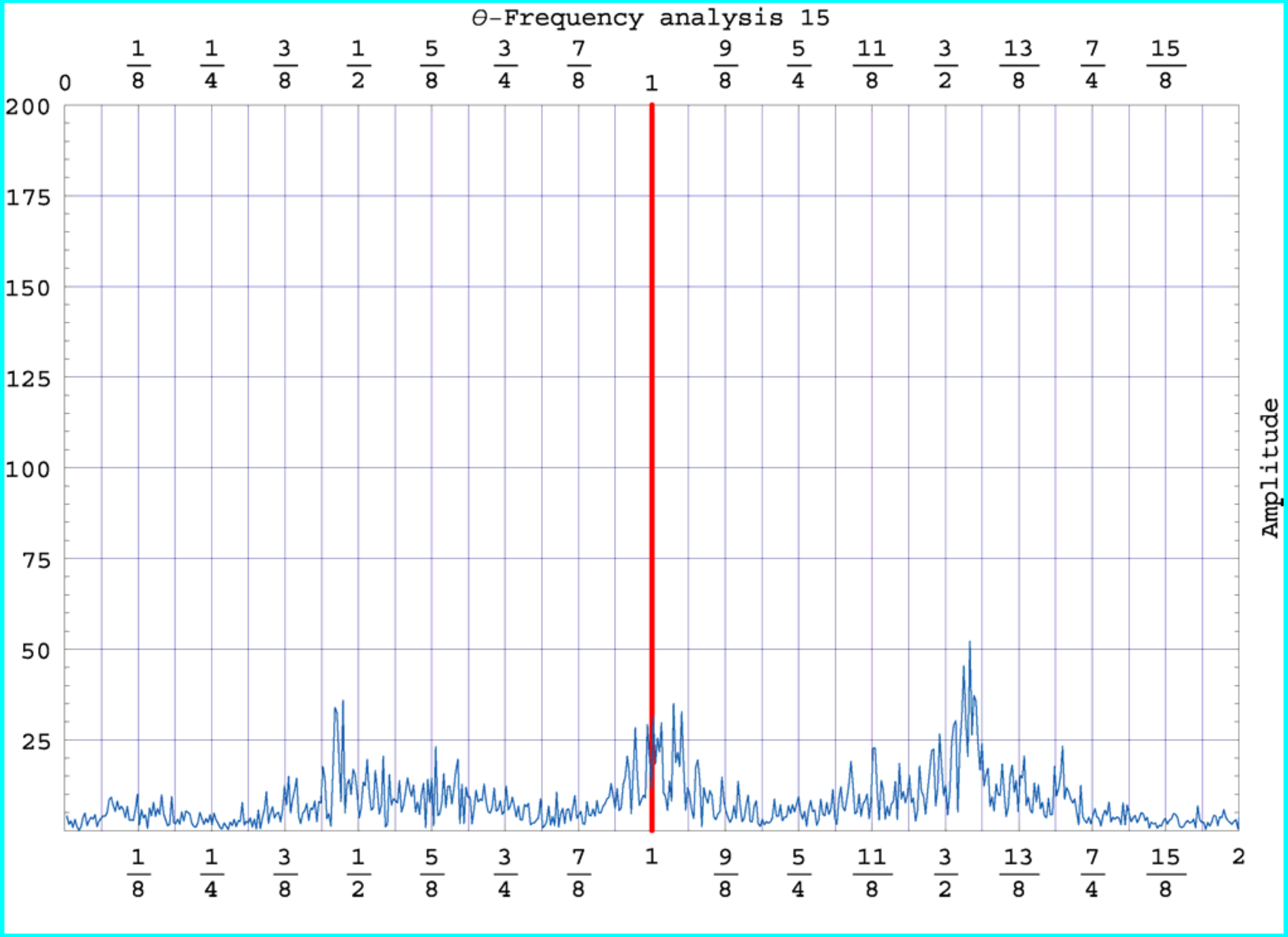
θ-Frequency analysis 11
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
9/8
5/4
11/8
3/2
13/8
7/4
15/8
2
150
125
100
75
50
25
0
Amplitude



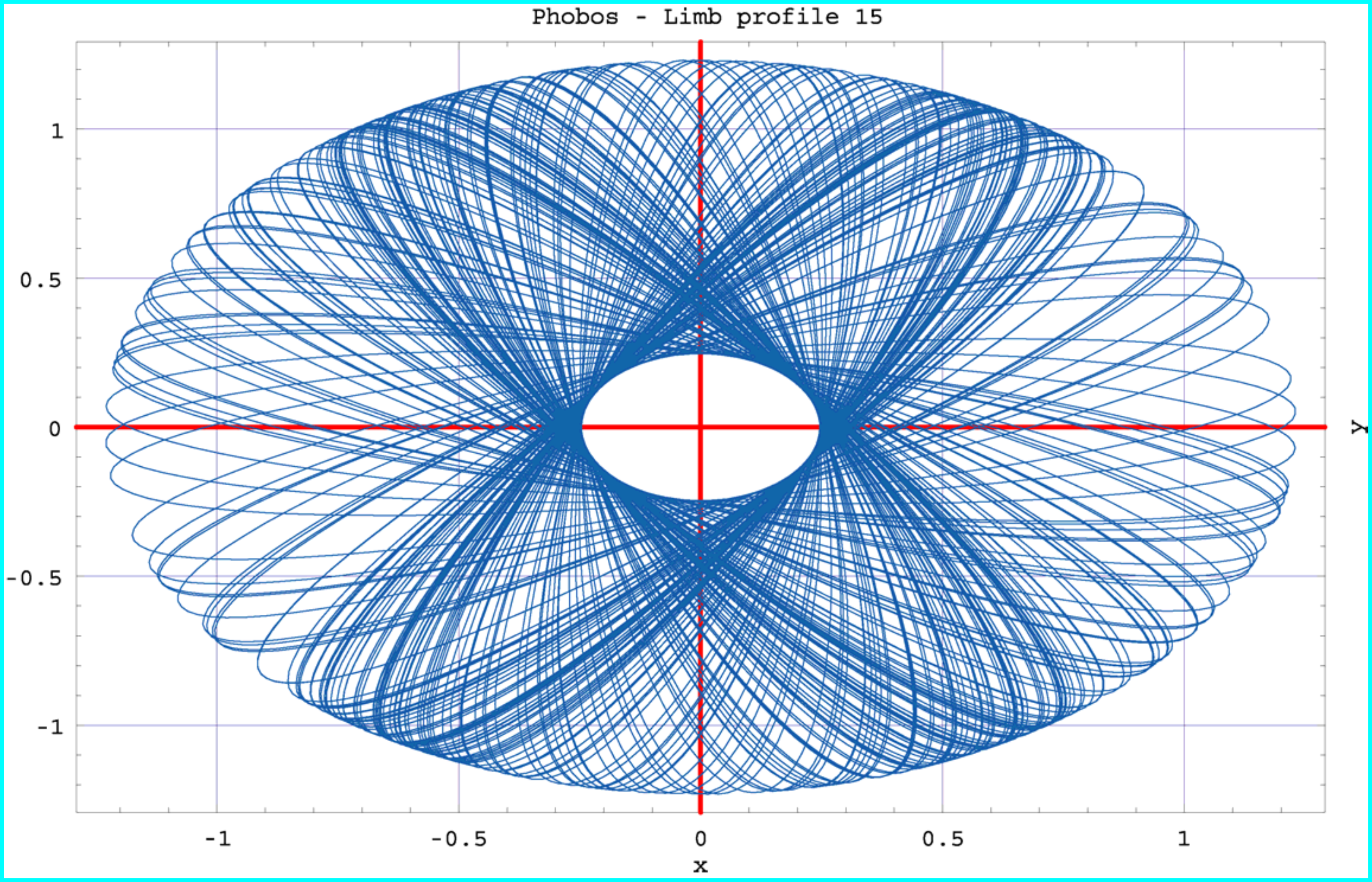
Phobos - Limb profile 11
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
x
y



Phase Space 15, Phobos
0
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2.5
2
1.5
1
0.5
0
-0.5
dθ/dt
π/4
π/2
3 π/4
π
5 π/4
3 π/2
7 π/4
2 π
θ



θ-Frequency analysis 15
Amplitude
0
1/8
1/4
3/8
1/2
5/8
3/4
7/8
1
9/8
5/4
11/8
3/2
13/8
7/4
15/8
2
200
175
150
125
100
75
50
25



Phobos - Limb profile 15
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
x
y

Lyapunov Exponents

$$\frac{d^2\theta}{dt^2}+\frac{3}{2}\frac{B-A}{C}n^2\left(\frac{a}{r}\right)^3\sin(2(\theta-f))=W$$

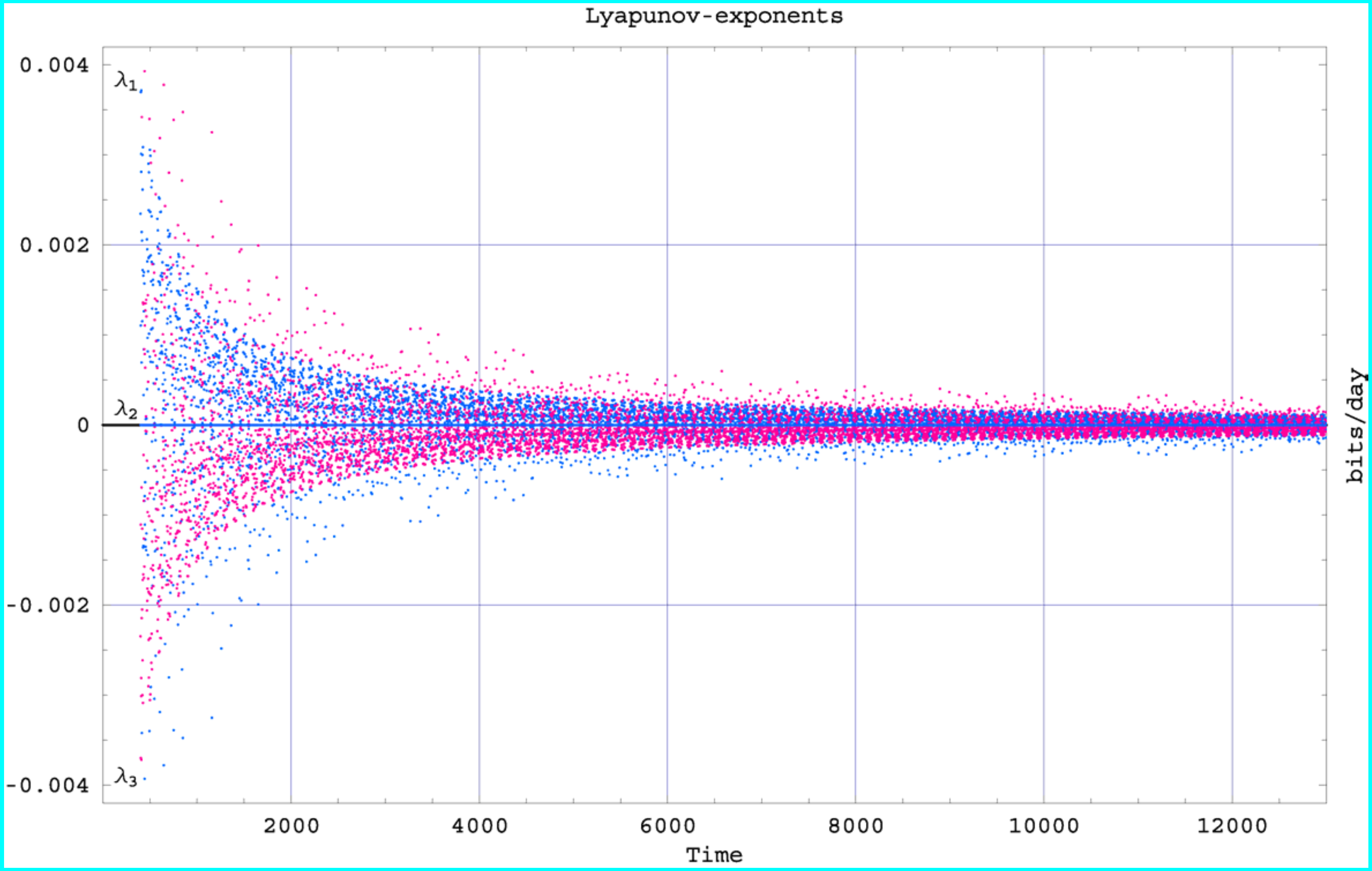
- Linearized Equations

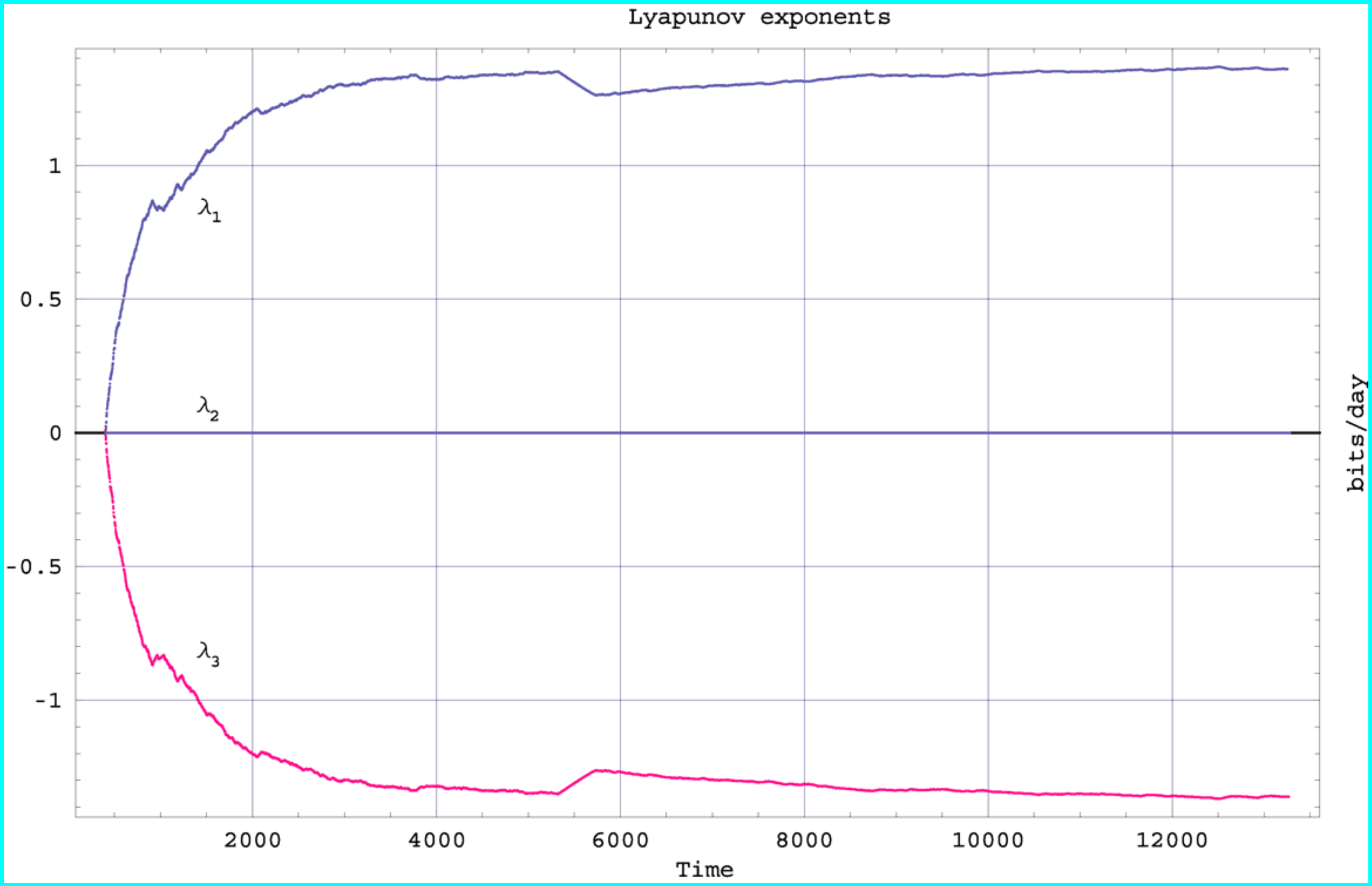
```
Tmp2 := y3 + C1 * Sin[y3] + C2 * Sin[2 * y3] + C3 * Sin[3 * y3];
Tmp := ((s * (1 - e^2)) / (1 + e * Cos[Tmp2]));

f1 = y2;
f2 = -(1 / 2) * ALPHA^2 * n^2 * s^3 * Tmp^-3 * Sin[2 * (y1 - Tmp2)];
f3 = 2 * Pi / T;

MatrixForm[{{Hold[D[f1, y1]], Hold[D[f1, y2]], Hold[D[f1, y3]]},
   {Hold[D[f2, y1]], Hold[D[f2, y2]], Hold[D[f2, y3]]},
   {Hold[D[f3, y1]], Hold[D[f3, y2]], Hold[D[f3, y3]]}} .
  {dx, dy, dz}]
```

$$\begin{pmatrix} dx\,Hold[\partial_{y1} f1] + dy\,Hold[\partial_{y2} f1] + dz\,Hold[\partial_{y3} f1] \\ dx\,Hold[\partial_{y1} f2] + dy\,Hold[\partial_{y2} f2] + dz\,Hold[\partial_{y3} f2] \\ dx\,Hold[\partial_{y1} f3] + dy\,Hold[\partial_{y2} f3] + dz\,Hold[\partial_{y3} f3] \end{pmatrix}$$

```
k = ReleaseHold[%];

i = Simplify[k]
```

$$\{dy,\ \frac{1}{4\ (-1+e^2)^3}\ (ALPHA^2\ n^2\ (1+e\,Cos[y3+C1\,Sin[y3]+C2\,Sin[2\,y3]+C3\,Sin[3\,y3]])^2$$
$$(-dz\ (1+C1\,Cos[y3]+2\,C2\,Cos[2\,y3]+3\,C3\,Cos[3\,y3])$$
$$(5\,e\,Cos[2\,y1-3\,y3-3\,C1\,Sin[y3]-3\,C2\,Sin[2\,y3]-3\,C3\,Sin[3\,y3]]+$$
$$4\,Cos[2\ (y1-y3-C1\,Sin[y3]-C2\,Sin[2\,y3]-C3\,Sin[3\,y3])]-$$
$$e\,Cos[2\,y1-y3-C1\,Sin[y3]-C2\,Sin[2\,y3]-C3\,Sin[3\,y3]])+$$
$$4\,dx\,Cos[2\ (y1-y3-C1\,Sin[y3]-C2\,Sin[2\,y3]-C3\,Sin[3\,y3])]$$
$$(1+e\,Cos[y3+C1\,Sin[y3]+C2\,Sin[2\,y3]+C3\,Sin[3\,y3]]))),$$
$$0\}$$


Lyapunov-exponents
0.004
0.002
0
-0.002
-0.004
λ_1
λ_2
λ_3
2000
4000
6000
8000
10000
12000
Time
bits/day



Lyapunov exponents
1
0.5
0
-0.5
-1
2000
4000
6000
8000
10000
12000
Time
bits/day
λ1
λ2
λ3